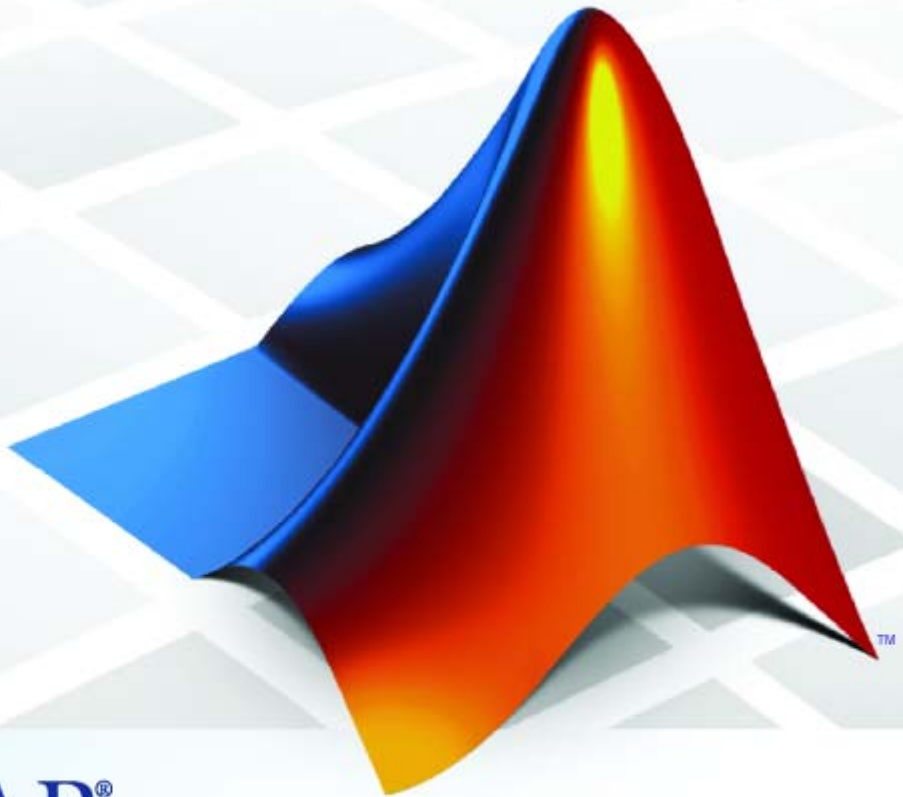


# Symbolic Math Toolbox™ 5

## User's Guide



MATLAB®

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# Introduction

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- “Accessing Symbolic Math Toolbox Functionality” on page 1-3

## Product Overview

Symbolic Math Toolbox™ software lets you to perform symbolic computations within the MATLAB® numeric environment. It provides tools for solving and manipulating symbolic math expressions and performing variable-precision arithmetic. The toolbox contains hundreds of symbolic functions that leverage the MuPAD® engine for a broad range of mathematical tasks such as:

- Differentiation
- Integration
- Linear algebraic operations
- Simplification
- Transforms
- Variable-precision arithmetic
- Equation solving

Symbolic Math Toolbox software also includes the MuPAD language, which is optimized for handling and operating on symbolic math expressions. In addition to covering common mathematical tasks, the libraries of MuPAD functions cover specialized areas such as number theory and combinatorics. You can extend the built-in functionality by writing custom symbolic functions and libraries in the MuPAD language.

If you have a version of Maple™ software compatible with the Symbolic Math Toolbox software that you are using, you can use that instead of MuPAD software. See “Choosing a Maple or MuPAD Engine” on page 4-34.

# Accessing Symbolic Math Toolbox Functionality

## Key Features

Symbolic Math Toolbox software provides a complete set of tools for symbolic computing that augments the numeric capabilities of MATLAB. The toolbox includes extensive symbolic functionality that you can access directly from the MATLAB command line or from the MuPAD Notebook Interface. You can extend the functionality available in the toolbox by writing custom symbolic functions or libraries in the MuPAD language.

## Working from MATLAB

You can access the Symbolic Math Toolbox functionality directly from the MATLAB Command Window. This environment lets you call functions using familiar MATLAB syntax.

The MATLAB Help browser presents the documentation that covers working from the MATLAB Command Window. To access the MATLAB Help browser, you can:

- Select **Help > Product Help**, and then select **Symbolic Math Toolbox** in the left pane
- Enter `doc` at the MATLAB command line

If you are a new user, begin with Chapter 2, “Getting Started”

## Working from MuPAD

Also you can access the Symbolic Math Toolbox functionality from the MuPAD Notebook Interface using the MuPAD language. The MuPAD Notebook Interface includes a symbol palette for accessing common MuPAD functions. All results are displayed in typeset math. You also can convert the results into MathML and TeX. You can embed graphics, animations, and descriptive text within your notebook.

An editor, debugger, and other programming utilities provide tools for authoring custom symbolic functions and libraries in the MuPAD language. The MuPAD language supports multiple programming styles including

imperative, functional, and object-oriented programming. The language treats variables as symbolic by default and is optimized for handling and operating on symbolic math expressions. You can call functions written in the MuPAD language from the MATLAB Command Window. For more information see “Calling MuPAD Functions at the MATLAB Command Line” on page 4-27

The MuPAD Help browser presents documentation covering the MuPAD Notebook Interface. To access the MuPAD Help browser :

- From the MuPAD Notebook Interface, select **Help > Open Help**
- From the MATLAB Command Window, enter `doc(symengine)`.

If you are a new user of the MuPAD Notebook Interface, read the Getting Started chapter of the MuPAD documentation.

There is also a MuPAD Tutorial PDF file available at  
[http://www.mathworks.com/access/helpdesk/...  
help/pdf\\_doc/symbolic/mupad\\_tutorial.pdf](http://www.mathworks.com/access/helpdesk/...help/pdf_doc/symbolic/mupad_tutorial.pdf) .

# Getting Started

---

- “Symbolic Objects” on page 2-2
- “Creating Symbolic Variables and Expressions” on page 2-6
- “Performing Symbolic Computations” on page 2-12
- “Assumptions for Symbolic Objects” on page 2-30

## Symbolic Objects

In this section...
“Overview” on page 2-2
“Symbolic Variables” on page 2-2
“Symbolic Numbers” on page 2-3

### Overview

Symbolic objects are a special MATLAB data type introduced by the Symbolic Math Toolbox software. They allow you to perform mathematical operations in the MATLAB workspace analytically, without calculating numeric values. You can use symbolic objects to perform a wide variety of analytical computations:

- Differentiation, including partial differentiation
- Definite and indefinite integration
- Taking limits, including one-sided limits
- Summation, including Taylor series
- Matrix operations
- Solving algebraic and differential equations
- Variable-precision arithmetic
- Integral transforms

Symbolic objects present symbolic variables, symbolic numbers, symbolic expressions and symbolic matrices.

### Symbolic Variables

To declare variables  $x$  and  $y$  as symbolic objects use the `syms` command:

```
syms x y
```



You can manipulate the symbolic objects according to the usual rules of mathematics. For example:

```
x + x + y
ans =
2*x + y
```

You also can create formal symbolic mathematical expressions and symbolic matrices. See “Creating Symbolic Variables and Expressions” on page 2-6 for more information.

## Symbolic Numbers

Symbolic Math Toolbox software also enables you to convert numbers to symbolic objects. To create a symbolic number, use the `sym` command:

```
a = sym('2')
```

If you create a symbolic number with 10 or fewer decimal digits, you can skip the quotes:

```
a = sym(2)
```

The following example illustrates the difference between a standard double-precision MATLAB data and the corresponding symbolic number. The MATLAB command

```
sqrt(2)
```

returns a double-precision floating-point number:

```
ans =
1.4142
```

On the other hand, if you calculate a square root of a symbolic number 2:

```
a = sqrt(sym(2))
```

you get the precise symbolic result:

```
a =
2^(1/2)
```

Symbolic results are not indented. Standard MATLAB double-precision results are indented. The difference in output form shows what type of data is presented as a result.

To evaluate a symbolic number numerically, use the `double` command:

```
double(a)

ans =
    1.4142
```

You also can create a rational fraction involving symbolic numbers:

```
sym(2) / sym(5)

ans =
    2/5
```

or more efficiently:

```
sym(2/5)

ans =
    2/5
```

MATLAB performs arithmetic on symbolic fractions differently than it does on standard numeric fractions. By default, MATLAB stores all numeric values as double-precision floating-point data. For example:

```
2/5 + 1/3

ans =
    0.7333
```

If you add the same fractions as symbolic objects, MATLAB finds their common denominator and combines them in the usual procedure for adding rational numbers:

```
sym(2/5) + sym(1/3)

ans =
    11/15
```

To learn more about symbolic representation of rational and decimal fractions, see “Estimating the Precision of Numeric to Symbolic Conversions” on page 2-17.

## Creating Symbolic Variables and Expressions

### In this section...

“Creating Symbolic Variables” on page 2-6

“Creating Symbolic Expressions” on page 2-7

“Creating Symbolic Objects with Identical Names” on page 2-8

“Creating a Matrix of Symbolic Variables” on page 2-9

“Creating a Matrix of Symbolic Numbers” on page 2-10

“Finding Symbolic Variables in Expressions and Matrices” on page 2-10

### Creating Symbolic Variables

The `sym` command creates symbolic variables and expressions. For example, the commands

```
x = sym('x');  
a = sym('alpha');
```

create a symbolic variable `x` with the value `x` assigned to it in the MATLAB workspace and a symbolic variable `a` with the value `alpha` assigned to it. An alternate way to create a symbolic object is to use the `syms` command:

```
syms x;  
a = sym('alpha');
```

You can use `sym` or `syms` to create symbolic variables. The `syms` command:

- Does not use parentheses and quotation marks: `syms x`
- Can create multiple objects with one call
- Serves best for creating individual single and multiple symbolic variables

The `sym` command:

- Requires parentheses and quotation marks: `x = sym('x')`. When creating a symbolic number with 10 or fewer decimal digits, you can skip the quotation marks: `f = sym(5)`.
- Creates one symbolic object with each call.
- Serves best for creating symbolic numbers and symbolic expressions.
- Serves best for creating symbolic objects in functions and scripts.

---

**Note** In Symbolic Math Toolbox, `pi` is a reserved word.

---

## Creating Symbolic Expressions

Suppose you want to use a symbolic variable to represent the golden ratio

$$\rho = \frac{1 + \sqrt{5}}{2}$$

The command

```
rho = sym('(1 + sqrt(5))/2');
```

achieves this goal. Now you can perform various mathematical operations on `rho`. For example,

```
f = rho^2 - rho - 1
```

returns

```
f =
(5^(1/2))/2 + 1/2)^2 - 5^(1/2)/2 - 3/2
```

Now suppose you want to study the quadratic function  $f = ax^2 + bx + c$ . One approach is to enter the command

```
f = sym('a*x^2 + b*x + c');
```

which assigns the symbolic expression  $ax^2 + bx + c$  to the variable `f`. However, in this case, Symbolic Math Toolbox software does not create variables corresponding to the terms of the expression: `a`, `b`, `c`, and `x`. To perform

symbolic math operations on  $f$ , you need to create the variables explicitly. A better alternative is to enter the commands

```
a = sym('a');  
b = sym('b');  
c = sym('c');  
x = sym('x');
```

or simply

```
syms a b c x
```

Then, enter

```
f = a*x^2 + b*x + c;
```

---

**Note** To create a symbolic expression that is a constant, you must use the `sym` command. Do not use `syms` command to create a symbolic expression that is a constant. For example, to create the expression whose value is 5, enter `f = sym(5)`. The command `f = 5` does *not* define  $f$  as a symbolic expression.

---

## Creating Symbolic Objects with Identical Names

If you set a variable equal to a symbolic expression, and then apply the `syms` command to the variable, MATLAB software removes the previously defined expression from the variable. For example,

```
syms a b;  
f = a + b
```

returns

```
f =  
a + b
```

If later you enter

```
syms f;  
f
```

then MATLAB removes the value  $a + b$  from the expression  $f$ :

```
f =
f
```

You can use the `syms` command to clear variables of definitions that you previously assigned to them in your MATLAB session. However, `syms` does not clear the following assumptions of the variables: complex, real, and positive. These assumptions are stored separately from the symbolic object. See “Deleting Symbolic Objects and Their Assumptions” on page 2-31 for more information.

## Creating a Matrix of Symbolic Variables

A circulant matrix has the property that each row is obtained from the previous one by cyclically permuting the entries one step forward. You can create the symbolic circulant matrix `A` whose elements are `a`, `b`, and `c`, using the commands:

```
syms a b c;
A = [a b c; c a b; b c a]

A =
[ a, b, c]
[ c, a, b]
[ b, c, a]
```

Since the matrix `A` is circulant, the sum of elements over each row and each column is the same. Find the sum of all the elements of the first row:

```
sum(A(1,:))

ans =
a + b + c
```

Check if the sum of the elements of the first row equals the sum of the elements of the second column:

```
sum(A(1,:)) == sum(A(:,2))
```

The sums are equal:

```
ans =
1
```

From this example, you can see that using symbolic objects is very similar to using regular MATLAB numeric objects.

## Creating a Matrix of Symbolic Numbers

A particularly effective use of `sym` is to convert a matrix from numeric to symbolic form. The command

```
A = hilb(3)
```

generates the 3-by-3 Hilbert matrix:

```
A =  
    1.0000    0.5000    0.3333  
    0.5000    0.3333    0.2500  
    0.3333    0.2500    0.2000
```

By applying `sym` to `A`

```
A = sym(A)
```

you can obtain the precise symbolic form of the 3-by-3 Hilbert matrix:

```
A =  
[ 1, 1/2, 1/3]  
[ 1/2, 1/3, 1/4]  
[ 1/3, 1/4, 1/5]
```

For more information on numeric to symbolic conversions see “Estimating the Precision of Numeric to Symbolic Conversions” on page 2-17.

## Finding Symbolic Variables in Expressions and Matrices

To determine what symbolic variables are present in an expression, use the `symvar` command. For example, given the symbolic expressions `f` and `g` defined by

```
syms a b n t x z;  
f = x^n;  
g = sin(a*t + b);
```



you can find the symbolic variables in `f` by entering:

```
symvar(f)
```

```
ans =  
[ n, x]
```

Similarly, you can find the symbolic variables in `g` by entering:

```
symvar(g)
```

```
ans =  
[ a, b, t]
```

## Performing Symbolic Computations

### In this section...

“Simplifying Symbolic Expressions” on page 2-12

“Substituting in Symbolic Expressions” on page 2-14

“Estimating the Precision of Numeric to Symbolic Conversions” on page 2-17

“Differentiating Symbolic Expressions” on page 2-19

“Integrating Symbolic Expressions” on page 2-21

“Solving Equations” on page 2-23

“Finding a Default Symbolic Variable” on page 2-25

“Creating Plots of Symbolic Functions” on page 2-25

### Simplifying Symbolic Expressions

Symbolic Math Toolbox provides a set of simplification functions allowing you to manipulate an output of a symbolic expression. For example, the following polynomial of the golden ratio rho

```
rho = sym('(1 + sqrt(5))/2');  
f = rho^2 - rho - 1
```

returns

```
f =  
(5^(1/2)/2 + 1/2)^2 - 5^(1/2)/2 - 3/2
```

You can simplify this answer by entering

```
simplify(f)
```

and get a very short answer:

```
ans =  
0
```

Symbolic simplification is not always so straightforward. There is no universal simplification function, because the meaning of a simplest representation of a symbolic expression cannot be defined clearly. Different problems require different forms of the same mathematical expression. Knowing what form is more effective for solving your particular problem, you can choose the appropriate simplification function.

For example, to show the order of a polynomial or symbolically differentiate or integrate a polynomial, use the standard polynomial form with all the parenthesis multiplied out and all the similar terms summed up. To rewrite a polynomial in the standard form, use the `expand` function:

```
syms x;
f = (x ^2- 1)*(x^4 + x^3 + x^2 + x + 1)*(x^4 - x^3 + x^2 - x + 1);
expand(f)

ans =
x^10 - 1
```

The `factor` simplification function shows the polynomial roots. If a polynomial cannot be factored over the rational numbers, the output of the `factor` function is the standard polynomial form. For example, to factor the third-order polynomial, enter:

```
syms x;
g = x^3 + 6*x^2 + 11*x + 6;
factor(g)

ans =
(x + 3)*(x + 2)*(x + 1)
```

The nested (Horner) representation of a polynomial is the most efficient for numerical evaluations:

```
syms x;
h = x^5 + x^4 + x^3 + x^2 + x;
horner(h)

ans =
x*(x*(x*(x*(x + 1) + 1) + 1) + 1) + 1)
```

For a list of Symbolic Math Toolbox simplification functions, see “Simplifications” on page 3-42.

## Substituting in Symbolic Expressions

### subs Command

You can substitute a numeric value for a symbolic variable or replace one symbolic variable with another using the `subs` command. For example, to substitute the value  $x = 2$  in the symbolic expression

```
syms x;  
f = 2*x^2 - 3*x + 1;
```

enter the command

```
subs(f, 2)  
  
ans =  
    3
```

### Substituting in Multivariate Expressions

When your expression contains more than one variable, you can specify the variable for which you want to make the substitution. For example, to substitute the value  $x = 3$  in the symbolic expression

```
syms x y;  
f = x^2*y + 5*x*sqrt(y);
```

enter the command

```
subs(f, x, 3)  
  
ans =  
    9*y + 15*y^(1/2)
```

### Substituting One Symbolic Variable for Another

You also can substitute one symbolic variable for another symbolic variable. For example to replace the variable  $y$  with the variable  $x$ , enter

```
subs(f, y, x)

ans =
x^3 + 5*x^(3/2)
```

## Substituting a Matrix into a Polynomial

You can also substitute a matrix into a symbolic polynomial with numeric coefficients. There are two ways to substitute a matrix into a polynomial: element by element and according to matrix multiplication rules.

**Element-by-Element Substitution.** To substitute a matrix at each element, use the `subs` command:

```
A = [1 2 3;4 5 6];
syms x; f = x^3 - 15*x^2 - 24*x + 350;
subs(f,A)

ans =
    312    250    170
     78    -20   -118
```

You can do element-by-element substitution for rectangular or square matrices.

**Substitution in a Matrix Sense.** If you want to substitute a matrix into a polynomial using standard matrix multiplication rules, a matrix must be square. For example, you can substitute the magic square `A` into a polynomial `f`:

**1** Create the polynomial:

```
syms x;
f = x^3 - 15*x^2 - 24*x + 350;
```

**2** Create the magic square matrix:

```
A = magic(3)

A =
     8     1     6
     3     5     7
```

```
4      9      2
```

- 3** Get a row vector containing the numeric coefficients of the polynomial  $f$ :

```
b = sym2poly(f)
```

```
b =  
1   -15   -24   350
```

- 4** Substitute the magic square matrix  $A$  into the polynomial  $f$ . Matrix  $A$  replaces all occurrences of  $x$  in the polynomial. The constant times the identity matrix  $\text{eye}(3)$  replaces the constant term of  $f$ :

```
A^3 - 15*A^2 - 24*A + 350*eye(3)
```

```
ans =  
-10    0    0  
  0   -10   0  
  0    0  -10
```

The `polyvalm` command provides an easy way to obtain the same result:

```
polyvalm(sym2poly(f),A)
```

```
ans =  
-10    0    0  
  0   -10   0  
  0    0  -10
```

### **Substituting the Elements of a Symbolic Matrix**

To substitute a set of elements in a symbolic matrix, also use the `subs` command. Suppose you want to replace some of the elements of a symbolic circulant matrix  $A$

```
syms a b c;  
A = [a b c; c a b; b c a]
```

```
A =  
[ a, b, c]  
[ c, a, b]  
[ b, c, a]
```

To replace the (2, 1) element of A with beta and the variable b throughout the matrix with variable alpha, enter

```
alpha = sym('alpha');
beta = sym('beta');
A(2,1) = beta;
A = subs(A,b,alpha)
```

The result is the matrix:

```
A =
[      a, alpha,      c]
[ beta,      a, alpha]
[ alpha,      c,      a]
```

For more information on the subs command see “Substitutions” on page 3-53.

## Estimating the Precision of Numeric to Symbolic Conversions

The sym command converts a numeric scalar or matrix to symbolic form. By default, the sym command returns a rational approximation of a numeric expression. For example, you can convert the standard double-precision variable into a symbolic object:

```
t = 0.1;
sym(t)

ans =
1/10
```

The technique for converting floating-point numbers is specified by the optional second argument, which can be 'f', 'r', 'e' or 'd'. The default option is 'r' that stands for rational approximation “Converting to Rational Symbolic Form” on page 2-18.

## Converting to Floating-Point Symbolic Form

The 'f' option to sym converts a double-precision floating-point number to a sum of two binary numbers. All values are represented as rational numbers  $N \cdot 2^e$ , where e and N are integers, and N is nonnegative. For example,

```
sym(t, 'f')
```

returns the symbolic floating-point representation:

```
ans =  
3602879701896397/36028797018963968
```

### **Converting to Rational Symbolic Form**

If you call `sym` command with the 'r' option

```
sym(t, 'r')
```

you get the results in the rational form:

```
ans =  
1/10
```

This is the default setting for the `sym` command. If you call this command without any option, you get the result in the same rational form:

```
sym(t)  
  
ans =  
1/10
```

### **Converting to Rational Symbolic Form with Machine Precision**

If you call the `sym` command with the option 'e', it returns the rational form of `t` plus the difference between the theoretical rational expression for `t` and its actual (machine) floating-point value in terms of `eps` (the floating-point relative accuracy):

```
sym(t, 'e')  
  
ans =  
eps/40 + 1/10
```

### **Converting to Decimal Symbolic Form**

If you call the `sym` command with the option 'd', it returns the decimal expansion of `t` up to the number of significant digits:



```
sym(t, 'd')
```

```
ans =  
0.100000000000000000555111512312578
```

By default, the `sym(t, 'd')` command returns a number with 32 significant digits. To change the number of significant digits, use the `digits` command:

```
digits(7);  
sym(t, 'd')
```

```
ans =  
0.1
```

## Differentiating Symbolic Expressions

With the Symbolic Math Toolbox software, you can find

- Derivatives of single-variable expressions
- Partial derivatives
- Second and higher order derivatives
- Mixed derivatives

For in-depth information on taking symbolic derivatives see “Differentiation” on page 3-2.

## Expressions with One Variable

To differentiate a symbolic expression, use the `diff` command. The following example illustrates how to take a first derivative of a symbolic expression:

```
syms x;  
f = sin(x)^2;  
diff(f)  
  
ans =  
2*cos(x)*sin(x)
```

## Partial Derivatives

For multivariable expressions, you can specify the differentiation variable. If you do not specify any variable, MATLAB chooses a default variable by the proximity to the letter *x*:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(f)  
  
ans =  
2*cos(x)*sin(x)
```

For the complete set of rules MATLAB applies for choosing a default variable, see “Finding a Default Symbolic Variable” on page 2-25.

To differentiate the symbolic expression *f* with respect to a variable *y*, enter:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(f, y)  
  
ans =  
(-2)*cos(y)*sin(y)
```

## Second Partial and Mixed Derivatives

To take a second derivative of the symbolic expression *f* with respect to a variable *y*, enter:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(f, y, 2)  
  
ans =  
2*sin(y)^2 - 2*cos(y)^2
```

You get the same result by taking derivative twice: `diff(diff(f, y))`. To take mixed derivatives, use two differentiation commands. For example:

```
syms x y;  
f = sin(x)^2 + cos(y)^2;  
diff(diff(f, y), x)
```

```
ans =
0
```

## Integrating Symbolic Expressions

You can perform symbolic integration including:

- Indefinite and definite integration
- Integration of multivariable expressions

For in-depth information on the `int` command including integration with real and complex parameters, see “Integration” on page 3-12.

### Indefinite Integrals of One-Variable Expressions

Suppose you want to integrate a symbolic expression. The first step is to create the symbolic expression:

```
syms x;
f = sin(x)^2;
```

To find the indefinite integral, enter

```
int(f)

ans =
x/2 - sin(2*x)/4
```

### Indefinite Integrals of Multivariable Expressions

If the expression depends on multiple symbolic variables, you can designate a variable of integration. If you do not specify any variable, MATLAB chooses a default variable by the proximity to the letter `x`:

```
syms x y n;
f = x^n + y^n;
int(f)

ans =
x*y^n + (x*x^n)/(n + 1)
```

For the complete set of rules MATLAB applies for choosing a default variable, see “Finding a Default Symbolic Variable” on page 2-25.

You also can integrate the expression  $f = x^n + y^n$  with respect to  $y$

```
syms x y n;  
f = x^n + y^n;  
int(f, y)  
  
ans =  
x^n*y + (y*y^n)/(n + 1)
```

If the integration variable is  $n$ , enter

```
syms x y n;  
f = x^n + y^n;  
int(f, n)  
  
ans =  
x^n/log(x) + y^n/log(y)
```

## Definite Integrals

To find a definite integral, pass the limits of integration as the final two arguments of the `int` function:

```
syms x y n;  
f = x^n + y^n;  
int(f, 1, 10)  
  
ans =  
piecewise([n = -1, log(10) + 9/y],...  
[n <> -1, (10^(n + 1) - 1)/(n + 1) + 9*y^n])
```

## If MATLAB Cannot Find a Closed Form of an Integral

If the `int` function cannot compute an integral, MATLAB issues a warning and returns the `int` function as an answer:

```
syms x y n;  
f = exp(x)^n + exp(y)^n;
```

```
int(f, n, 1, 10)
```

Warning: Explicit integral could not be found.

```
ans =
int(exp(x)^n + exp(y)^n, n = 1..10)
```

## Solving Equations

You can solve different types of symbolic equations including:

- Algebraic equations with one symbolic variable
- Algebraic equations with several symbolic variables
- Systems of algebraic equations

For in-depth information on solving symbolic equations including differential equations, see “Solving Equations” on page 3-93.

### Algebraic Equations with One Symbolic Variable

You can find the values of variable  $x$  for which the following expression is equal to zero:

```
syms x;
solve(x^3 - 6*x^2 + 11*x - 6)

ans =
1
2
3
```

By default, the `solve` command assumes that the right-side of the equation is equal to zero. If you want to solve an equation with a nonzero right part, use quotation marks around the equation:

```
syms x;
solve('x^3 - 6*x^2 + 11*x - 5 = 1')

ans =
1
2
```

3

### **Algebraic Equations with Several Symbolic Variables**

If an equation contains several symbolic variables, you can designate a variable for which this equation should be solved. For example, you can solve the multivariable equation:

```
syms x y;  
f = 6*x^2 - 6*x^2*y + x*y^2 - x*y + y^3 - y^2;
```

with respect to a symbolic variable y:

```
solve(f, y)  
  
ans =  
     1  
    2*x  
   (-3)*x
```

If you do not specify any variable, you get the solution of an equation for the alphabetically closest to x variable. For the complete set of rules MATLAB applies for choosing a default variable see “Finding a Default Symbolic Variable” on page 2-25.

### **Systems of Algebraic Equations**

You also can solve systems of equations. For example:

```
syms x y z;  
[x, y, z] = solve('z = 4*x', 'x = y', 'z = x^2 + y^2')  
  
x =  
     0  
     2  
  
y =  
     0  
     2  
  
z =  
     0
```

## Finding a Default Symbolic Variable

When performing substitution, differentiation, or integration, if you do not specify a variable to use, MATLAB uses a *default* variable. The default variable is basically the one closest alphabetically to  $x$ . To find which variable is chosen as a default variable, use the `symvar(expression, 1)` command.

For example:

```
syms s t;  
g = s + t;  
symvar(g, 1)
```

```
ans =  
t
```

```
syms sx tx;  
g = sx + tx;  
symvar(g, 1)
```

```
ans =  
tx
```

For more information on choosing the default symbolic variable, see the `symvar` command.

## Creating Plots of Symbolic Functions

You can create different types of graphs including:

- Plots of explicit functions
- Plots of implicit functions
- 3-D parametric plots
- Surface plots

See “Pedagogical and Graphical Applications” on page 5-6 for in-depth coverage of Symbolic Math Toolbox graphics and visualization tools.

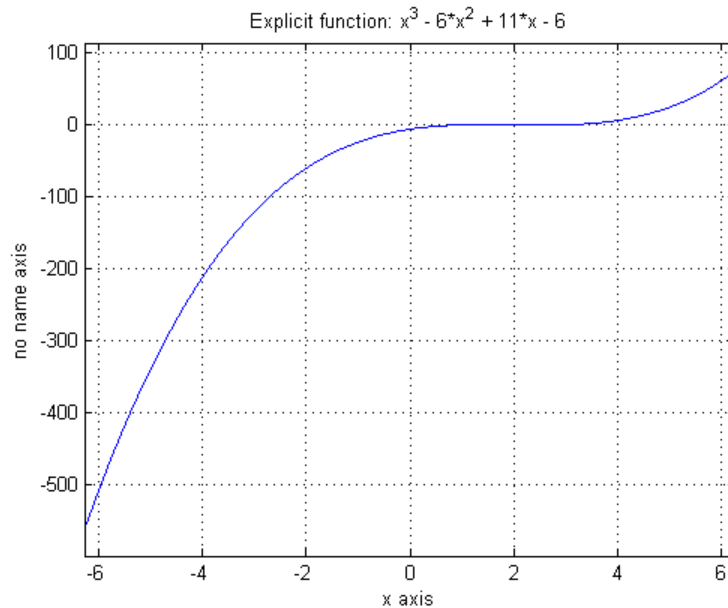
## Explicit Function Plot

The simplest way to create a plot is to use the `ezplot` command:

```
syms x;  
ezplot(x^3 - 6*x^2 + 11*x - 6);  
hold on;
```

The `hold on` command retains the existing plot allowing you to add new elements and change the appearance of the plot. For example, now you can change the names of the axes and add a new title and grid lines. When you finish working with the current plot, enter the `hold off` command:

```
xlabel('x axis');  
ylabel('no name axis');  
title('Explicit function: x^3 - 6*x^2 + 11*x - 6');  
grid on;  
hold off
```

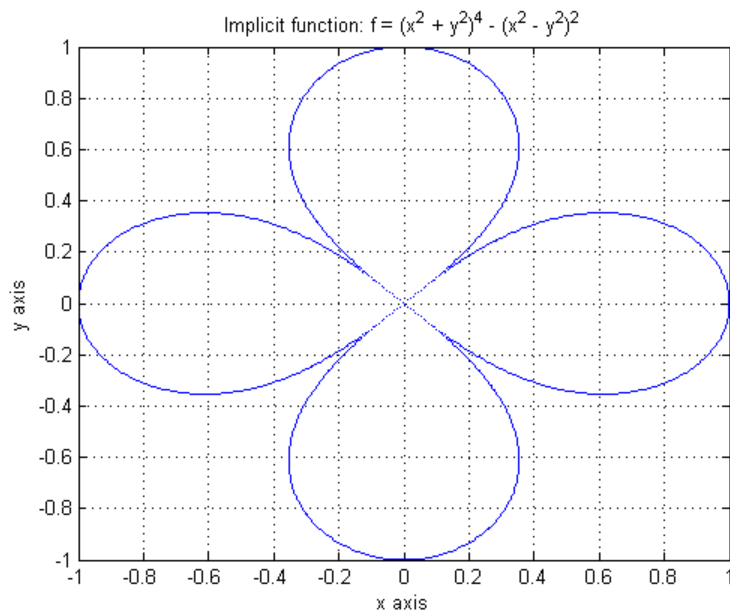




## Implicit Function Plot

You can plot implicitly defined functions. For example, create a plot for the following implicit function over the domain  $-1 < x < 1$ :

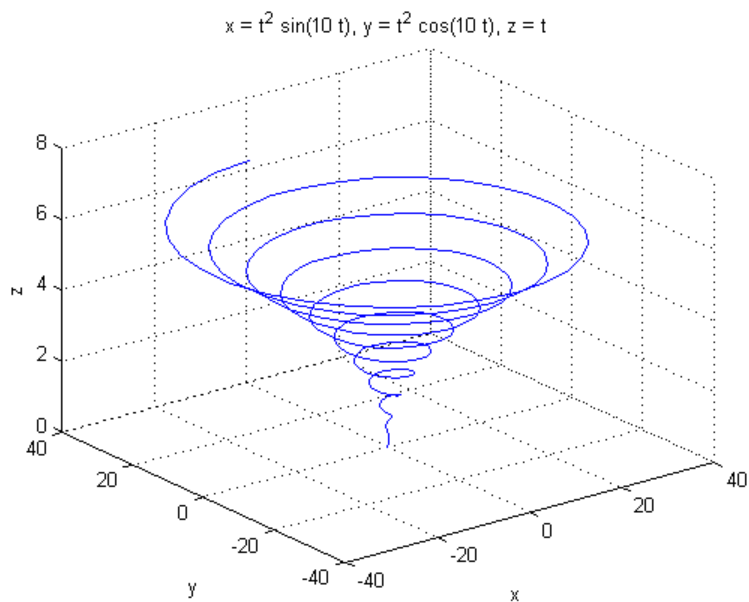
```
syms x y;  
f = (x^2 + y^2)^4 - (x^2 - y^2)^2;  
ezplot(f, [-1 1]);  
hold on;  
xlabel('x axis');  
ylabel('y axis');  
title('Implicit function: f = (x^2 + y^2)^4 - (x^2 - y^2)^2');  
grid on;  
hold off
```



### 3-D Plot

3-D graphics is also available in Symbolic Math Toolbox . To create a 3-D plot, use the `ezplot3` command. For example:

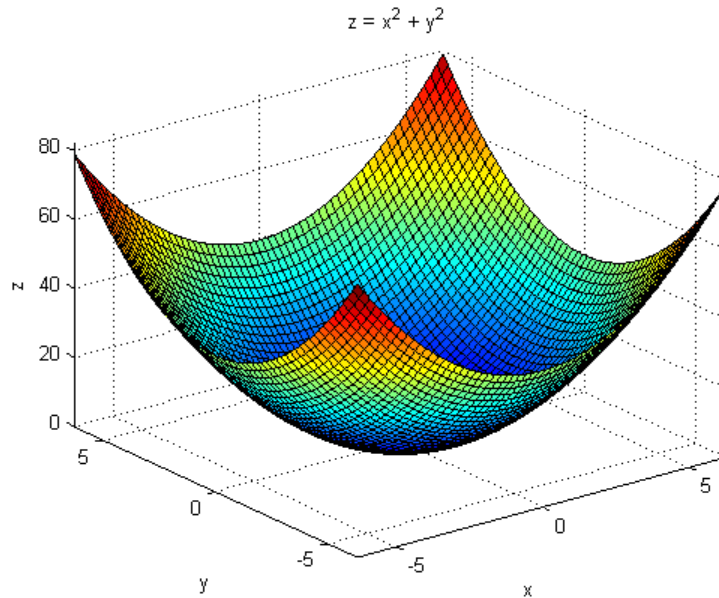
```
syms t;  
ezplot3(t^2*sin(10*t), t^2*cos(10*t), t);
```



## Surface Plot

If you want to create a surface plot, use the `ezsurf` command. For example, to plot a paraboloid  $z = x^2 + y^2$ , enter:

```
syms x y;  
ezsurf(x^2 + y^2);  
hold on;  
zlabel('z');  
title('z = x^2 + y^2');  
hold off
```



## Assumptions for Symbolic Objects

### In this section...

“Default Assumption” on page 2-30

“Setting Assumptions for Symbolic Variables” on page 2-30

“Deleting Symbolic Objects and Their Assumptions” on page 2-31

### Default Assumption

In Symbolic Math Toolbox, symbolic variables are single complex variables by default. For example, if you declare  $z$  as a symbolic variable:

```
syms z
```

MATLAB assumes  $z$  is a complex variable. You can always check if a symbolic variable is assumed to be complex or real by entering `conj` command. If `conj(x) == x` returns 1,  $x$  is a real variable:

```
z == conj(z)

ans =
    0
```

### Setting Assumptions for Symbolic Variables

The `sym` and `syms` commands allow you to set up assumptions for symbolic variables. For example, create the real symbolic variables  $x$  and  $y$  and the positive symbolic variable  $z$ :

```
x = sym('x', 'real');
y = sym('y', 'real');
z = sym('z', 'positive');
```

or more efficiently

```
syms x y real;
syms z positive;
```

There are two assumptions you can assign to a symbolic object within the `sym` command: real and positive. Together with the default complex property of a

symbolic variable, it gives you three choices for an assumption for a symbolic variable: complex, real, and positive.

## Deleting Symbolic Objects and Their Assumptions

When you declare  $x$  to be real with the command

```
syms x real
```

you create a symbolic object  $x$  and the assumption that the object is real. Symbolic objects and their assumptions are stored separately. When you delete a symbolic object from the MATLAB workspace

```
clear x
```

the assumption that  $x$  is real stays in symbolic engine. If you declare a new symbolic variable  $x$  later, it inherits the assumption that  $x$  is real instead of getting a default assumption. If later you solve an equation and simplify an expression with the symbolic variable  $x$ , you could get incomplete results. For example, the assumption that  $x$  is real causes the polynomial  $x^2+1$  to have no roots:

```
syms x real;
clear x;
syms x;
solve(x^2+1)
```

```
Warning: Explicit solution could not be found.
> In solve at 81
```

```
ans =
[ empty sym ]
```

The complex roots of this polynomial disappear because the symbolic variable  $x$  still has the assumption that  $x$  is real stored in the symbolic engine. To clear the assumption, enter

```
syms x clear
```

After you clear the assumption, the symbolic object stays in the MATLAB workspace. If you want to remove both the symbolic object and its assumption, use two subsequent commands:

**1** To clear the assumption, enter

```
syms x clear
```

**2** To delete the symbolic object, enter

```
clear x
```

For more information on clearing symbolic variables, see “Clearing Assumptions and Resetting the Symbolic Engine” on page 4-29.

# Using Symbolic Math Toolbox Software

---

This section explains how to use Symbolic Math Toolbox software to perform many common mathematical operations. The section covers the following topics:

- “Calculus” on page 3-2
- “Simplifications and Substitutions” on page 3-42
- “Variable-Precision Arithmetic” on page 3-60
- “Linear Algebra” on page 3-66
- “Solving Equations” on page 3-93
- “Integral Transforms and Z-Transforms” on page 3-102
- “Special Functions of Applied Mathematics” on page 3-120
- “Generating Code from Symbolic Expressions” on page 3-129

## Calculus

### In this section...

“Differentiation” on page 3-2

“Limits” on page 3-8

“Integration” on page 3-12

“Symbolic Summation” on page 3-19

“Taylor Series” on page 3-20

“Calculus Example” on page 3-22

“Extended Calculus Example” on page 3-30

## Differentiation

To illustrate how to take derivatives using Symbolic Math Toolbox software, first create a symbolic expression:

```
syms x
f = sin(5*x)
```

The command

```
diff(f)
```

differentiates  $f$  with respect to  $x$ :

```
ans =
5*cos(5*x)
```

As another example, let

```
g = exp(x)*cos(x)
```

where  $\exp(x)$  denotes  $e^x$ , and differentiate  $g$ :

```
diff(g)
ans =
exp(x)*cos(x) - exp(x)*sin(x)
```



To take the second derivative of  $g$ , enter

```
diff(g,2)
ans =
(-2)*exp(x)*sin(x)
```

You can get the same result by taking the derivative twice:

```
diff(diff(g))
ans =
(-2)*exp(x)*sin(x)
```

In this example, MATLAB software automatically simplifies the answer. However, in some cases, MATLAB might not simply an answer, in which case you can use the `simplify` command. For an example of this, see “More Examples” on page 3-5.

Note that to take the derivative of a constant, you must first define the constant as a symbolic expression. For example, entering

```
c = sym('5');
diff(c)
```

returns

```
ans =
0
```

If you just enter

```
diff(5)
```

MATLAB returns

```
ans =
[]
```

because 5 is not a symbolic expression.

### **Derivatives of Expressions with Several Variables**

To differentiate an expression that contains more than one symbolic variable, you must specify the variable that you want to differentiate with respect to.

The `diff` command then calculates the partial derivative of the expression with respect to that variable. For example, given the symbolic expression

```
syms s t
f = sin(s*t)
```

the command

```
diff(f,t)
```

calculates the partial derivative  $\partial f / \partial t$ . The result is

```
ans =
s*cos(s*t)
```

To differentiate `f` with respect to the variable `s`, enter

```
diff(f,s)
```

which returns:

```
ans =
t*cos(s*t)
```

If you do not specify a variable to differentiate with respect to, MATLAB chooses a default variable. Basically, the default variable is the letter closest to `x` in the alphabet. See the complete set of rules in “Finding a Default Symbolic Variable” on page 2-25. In the preceding example, `diff(f)` takes the derivative of `f` with respect to `t` because the letter `t` is closer to `x` in the alphabet than the letter `s` is. To determine the default variable that MATLAB differentiates with respect to, use the `symvar` command:

```
symvar(f, 1)
```

```
ans =
t
```

To calculate the second derivative of `f` with respect to `t`, enter

```
diff(f, t, 2)
```

which returns

```
ans =
-s^2*sin(s*t)
```

Note that `diff(f, 2)` returns the same answer because `t` is the default variable.

### More Examples

To further illustrate the `diff` command, define `a`, `b`, `x`, `n`, `t`, and `theta` in the MATLAB workspace by entering

```
syms a b x n t theta
```

The table below illustrates the results of entering `diff(f)`.

<b>f</b>	<b>diff(f)</b>
<pre>syms x n; f = x^n;</pre>	<pre>diff(f)  ans = n*x^(n - 1)</pre>
<pre>syms a b t; f = sin(a*t + b);</pre>	<pre>diff(f)  ans = a*cos(b + a*t)</pre>
<pre>syms theta; f = exp(i*theta);</pre>	<pre>diff(f)  ans = i*exp(i*theta)</pre>

To differentiate the Bessel function of the first kind, `besselj(nu, z)`, with respect to `z`, type

```
syms nu z
b = besselj(nu, z);
db = diff(b)
```

which returns

```
db =  
(nu*besselj(nu, z))/z - besselj(nu + 1, z)
```

The `diff` function can also take a symbolic matrix as its input. In this case, the differentiation is done element-by-element. Consider the example

```
syms a x  
A = [cos(a*x), sin(a*x); -sin(a*x), cos(a*x)]
```

which returns

```
A =  
[ cos(a*x), sin(a*x)]  
[ -sin(a*x), cos(a*x)]
```

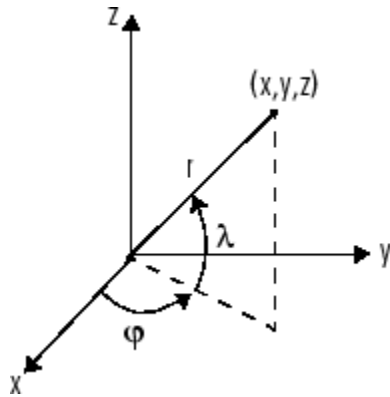
The command

```
diff(A)
```

returns

```
ans =  
[ -a*sin(a*x), a*cos(a*x)]  
[ -a*cos(a*x), -a*sin(a*x)]
```

You can also perform differentiation of a vector function with respect to a vector argument. Consider the transformation from Euclidean  $(x, y, z)$  to spherical  $(r, \lambda, \varphi)$  coordinates as given by  $x = r \cos \lambda \cos \varphi$ ,  $y = r \cos \lambda \sin \varphi$ , and  $z = r \sin \lambda$ . Note that  $\lambda$  corresponds to elevation or latitude while  $\varphi$  denotes azimuth or longitude.



To calculate the Jacobian matrix,  $J$ , of this transformation, use the `jacobian` function. The mathematical notation for  $J$  is

$$J = \frac{\partial(x, y, z)}{\partial(r, \lambda, \varphi)}.$$

For the purposes of toolbox syntax, use `l` for  $\lambda$  and `f` for  $\varphi$ . The commands

```
syms r l f
x = r*cos(l)*cos(f); y = r*cos(l)*sin(f); z = r*sin(l);
J = jacobian([x; y; z], [r l f])
```

return the Jacobian

```
J =
[ cos(f)*cos(l), -r*cos(f)*sin(l), -r*cos(l)*sin(f)]
[ cos(l)*sin(f), -r*sin(f)*sin(l),  r*cos(f)*cos(l)]
[          sin(l),          r*cos(l),          0]
```

and the command

```
detJ = simple(det(J))
```

returns

```
detJ =
-r^2*cos(l)
```

The arguments of the `jacobian` function can be column or row vectors. Moreover, since the determinant of the Jacobian is a rather complicated trigonometric expression, you can use the `simple` command to make trigonometric substitutions and reductions (simplifications). The section “Simplifications and Substitutions” on page 3-42 discusses simplification in more detail.

A table summarizing `diff` and `jacobian` follows.

<b>Mathematical Operator</b>	<b>MATLAB Command</b>
$\frac{df}{dx}$	<code>diff(f)</code> or <code>diff(f, x)</code>
$\frac{df}{da}$	<code>diff(f, a)</code>
$\frac{d^2f}{db^2}$	<code>diff(f, b, 2)</code>
$J = \frac{\partial(r,t)}{\partial(u,v)}$	<code>J = jacobian([r; t],[u; v])</code>

### Limits

The fundamental idea in calculus is to make calculations on functions as a variable “gets close to” or approaches a certain value. Recall that the definition of the derivative is given by a limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

provided this limit exists. Symbolic Math Toolbox software enables you to calculate the limits of functions directly. The commands

```
syms h n x
limit((cos(x+h) - cos(x))/h, h, 0)
```

which return

```
ans =  
-sin(x)
```

and

```
limit((1 + x/n)^n, n, inf)
```

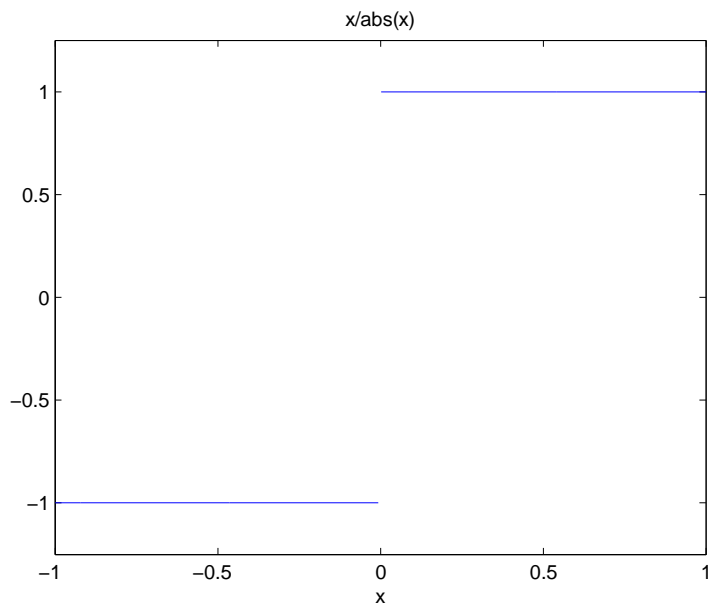
which returns

```
ans =  
exp(x)
```

illustrate two of the most important limits in mathematics: the derivative (in this case of  $\cos(x)$ ) and the exponential function.

### One-Sided Limits

You can also calculate one-sided limits with Symbolic Math Toolbox software. For example, you can calculate the limit of  $x/|x|$ , whose graph is shown in the following figure, as  $x$  approaches 0 from the left or from the right.



To calculate the limit as  $x$  approaches 0 from the left,

$$\lim_{x \rightarrow 0^-} \frac{x}{|x|},$$

enter

```
syms x;  
limit(x/abs(x), x, 0, 'left')
```

This returns

```
ans =  
-1
```

To calculate the limit as  $x$  approaches 0 from the right,

$$\lim_{x \rightarrow 0^+} \frac{x}{|x|} = 1,$$



enter

```
syms x;
limit(x/abs(x), x, 0, 'right')
```

This returns

```
ans =
1
```

Since the limit from the left does not equal the limit from the right, the two-sided limit does not exist. In the case of undefined limits, MATLAB returns NaN (not a number). For example,

```
syms x;
limit(x/abs(x), x, 0)
```

returns

```
ans =
NaN
```

Observe that the default case, `limit(f)` is the same as `limit(f,x,0)`. Explore the options for the `limit` command in this table, where `f` is a function of the symbolic object `x`.

<b>Mathematical Operation</b>	<b>MATLAB Command</b>
$\lim_{x \rightarrow 0} f(x)$	<code>limit(f)</code>
$\lim_{x \rightarrow a} f(x)$	<code>limit(f, x, a)</code> or <code>limit(f, a)</code>
$\lim_{x \rightarrow a^-} f(x)$	<code>limit(f, x, a, 'left')</code>
$\lim_{x \rightarrow a^+} f(x)$	<code>limit(f, x, a, 'right')</code>

## Integration

If  $f$  is a symbolic expression, then

$$\text{int}(f)$$

attempts to find another symbolic expression,  $F$ , so that  $\text{diff}(F) = f$ . That is,  $\text{int}(f)$  returns the indefinite integral or antiderivative of  $f$  (provided one exists in closed form). Similar to differentiation,

$$\text{int}(f, v)$$

uses the symbolic object  $v$  as the variable of integration, rather than the variable determined by `symvar`. See how `int` works by looking at this table.

Mathematical Operation	MATLAB Command
$\int x^n dx = \begin{cases} \log(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{otherwise.} \end{cases}$	<code>int(x^n)</code> or <code>int(x^n, x)</code>
$\int_0^{\pi/2} \sin(2x) dx = 1$	<code>int(sin(2*x), 0, pi/2)</code> or <code>int(sin(2*x), x, 0, pi/2)</code>
$g = \cos(at + b)$ $\int g(t) dt = \sin(at + b) / a$	<code>g = cos(a*t + b)</code> <code>int(g)</code> or <code>int(g, t)</code>
$\int J_1(z) dz = -J_0(z)$	<code>int(besselj(1, z))</code> or <code>int(besselj(1, z), z)</code>

In contrast to differentiation, symbolic integration is a more complicated task. A number of difficulties can arise in computing the integral:

- The antiderivative,  $F$ , may not exist in closed form.
- The antiderivative may define an unfamiliar function.
- The antiderivative may exist, but the software can't find it.

- The software could find the antiderivative on a larger computer, but runs out of time or memory on the available machine.

Nevertheless, in many cases, MATLAB can perform symbolic integration successfully. For example, create the symbolic variables

```
syms a b theta x y n u z
```

The following table illustrates integration of expressions containing those variables.

<b>f</b>	<b>int(f)</b>
<pre>syms x n; f = x^n;</pre>	<pre>int(f)  ans = piecewise([n = -1, log(x)], [n &lt;&gt; -1, x^(n + 1)/(n + 1)])</pre>
<pre>syms y; f = y^(-1);</pre>	<pre>int(f)  ans = log(y)</pre>
<pre>syms x n; f = n^x;</pre>	<pre>int(f)  ans = n^x/log(n)</pre>
<pre>syms a b theta; f = sin(a*theta+b);</pre>	<pre>int(f)  ans = -cos(b + a*theta)/a</pre>

<b>f</b>	<b>int(f)</b>
<pre>syms u; f = 1/(1+u^2);</pre>	<pre>int(f)  ans = atan(u)</pre>
<pre>syms x; f = exp(-x^2);</pre>	<pre>int(f)  ans = (pi^(1/2)*erf(x))/2</pre>

In the last example,  $\exp(-x^2)$ , there is no formula for the integral involving standard calculus expressions, such as trigonometric and exponential functions. In this case, MATLAB returns an answer in terms of the error function `erf`.

If MATLAB is unable to find an answer to the integral of a function `f`, it just returns `int(f)`.

Definite integration is also possible.

<b>Definite Integral</b>	<b>Command</b>
$\int_a^b f(x)dx$	<pre>int(f, a, b)</pre>
$\int_a^b f(v)dv$	<pre>int(f, v, a, b)</pre>

Here are some additional examples.

<b>f</b>	<b>a, b</b>	<b>int(f, a, b)</b>
<pre>syms x; f = x^7;</pre>	<pre>a = 0; b = 1;</pre>	<pre>int(f, a, b) ans = 1/8</pre>
<pre>syms x; f = 1/x;</pre>	<pre>a = 1; b = 2;</pre>	<pre>int(f, a, b) ans = log(2)</pre>
<pre>syms x; f = log(x)*sqrt(x);</pre>	<pre>a = 0; b = 1;</pre>	<pre>int(f, a, b) ans = -4/9</pre>
<pre>syms x; f = exp(-x^2);</pre>	<pre>a = 0; b = inf;</pre>	<pre>int(f, a, b) ans = pi^(1/2)/2</pre>
<pre>syms z; f = besselj(1,z)^2;</pre>	<pre>a = 0; b = 1;</pre>	<pre>int(f, a, b) ans = hypergeom([3/2, 3/2], [2, 5/2, 3], -1)/12</pre>

For the Bessel function (`besselj`) example, it is possible to compute a numerical approximation to the value of the integral, using the `double` function. The commands

```
syms z
a = int(besselj(1,z)^2,0,1)
```

```
return
```

```
a =  
hypergeom([3/2, 3/2], [2, 5/2, 3], -1)/12
```

and the command

```
a = double(a)
```

returns

```
a =  
0.0717
```

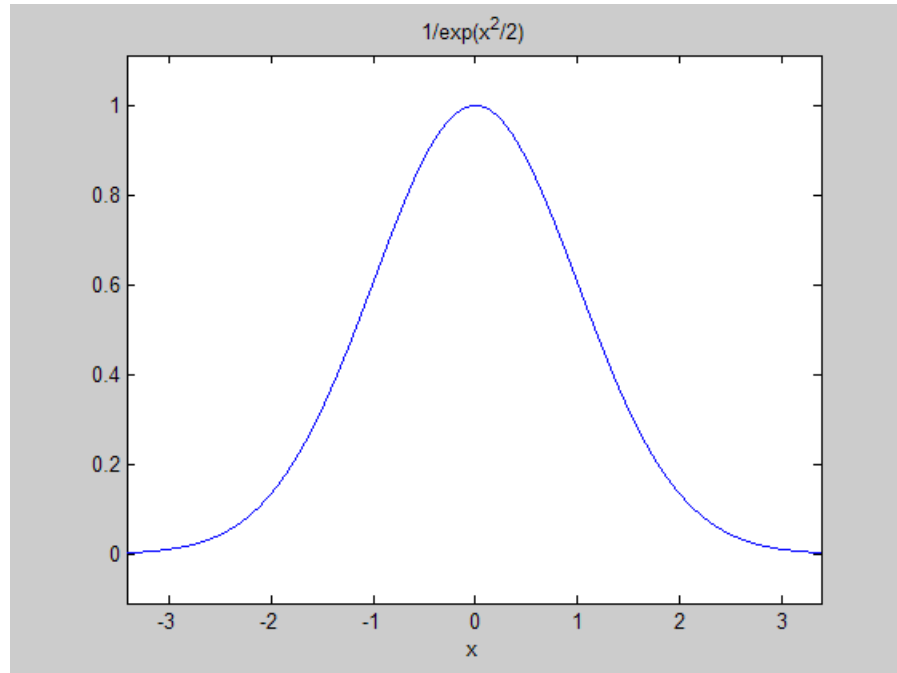
### **Integration with Real Parameters**

One of the subtleties involved in symbolic integration is the “value” of various parameters. For example, if  $a$  is any positive real number, the expression

$$e^{-ax^2}$$

is the positive, bell shaped curve that tends to 0 as  $x$  tends to  $\pm\infty$ . You can create an example of this curve, for  $a = 1/2$ , using the following commands:

```
syms x  
a = sym(1/2);  
f = exp(-a*x^2);  
ezplot(f)
```



However, if you try to calculate the integral

$$\int_{-\infty}^{\infty} e^{-ax^2} dx$$

without assigning a value to  $a$ , MATLAB assumes that  $a$  represents a complex number, and therefore returns a piecewise answer that depends on the argument of  $a$ . If you are only interested in the case when  $a$  is a positive real number, you can calculate the integral as follows:

```
syms a positive;
```

The argument `positive` in the `syms` command restricts  $a$  to have positive values. Now you can calculate the preceding integral using the commands

```
syms x;  
f = exp(-a*x^2);
```

```
int(f, x, -inf, inf)
```

This returns

```
ans =  
pi^(1/2)/a^(1/2)
```

### Integration with Complex Parameters

To calculate the integral

$$\int_{-\infty}^{\infty} \frac{1}{a^2 + x^2} dx$$

for complex values of *a*, enter

```
syms a x clear  
f = 1/(a^2 + x^2);  
F = int(f, x, -inf, inf)
```

`syms` is used with the `clear` option to clear the real property that was assigned to *a* in the preceding example — see “Deleting Symbolic Objects and Their Assumptions” on page 2-31.

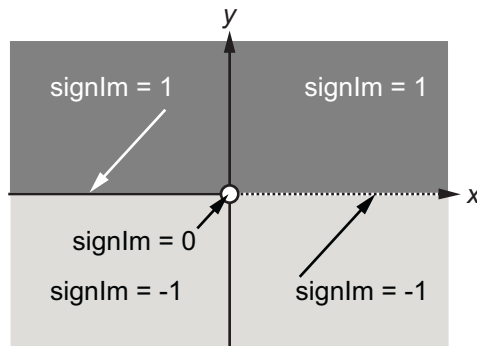
The preceding commands produce the complex output

```
F =  
(pi*signIm(i/a))/a
```

The function `signIm` is defined as:

$$\text{signIm}(z) = \begin{cases} 1 & \text{if } \text{Im}(z) > 0, \text{ or } \text{Im}(z) = 0 \text{ and } z < 0 \\ 0 & \text{if } z = 0 \\ -1 & \text{otherwise.} \end{cases}$$





To evaluate  $F$  at  $a = 1 + i$ , enter

```
g = subs(F, 1 + i)
```

```
g =  
pi/(2*i)^(1/2)
```

```
double(g)
```

```
ans =  
1.5708 - 1.5708i
```

## Symbolic Summation

You can compute symbolic summations, when they exist, by using the `symsum` command. For example, the p-series

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

sums to  $\pi^2/6$ , while the geometric series

$$1 + x + x^2 + \dots$$

sums to  $1/(1-x)$ , provided  $|x| < 1$ . These summations are demonstrated below:

```
syms x k  
s1 = symsum(1/k^2, 1, inf)
```

```
s2 = symsum(x^k, k, 0, inf)

s1 =
pi^2/6

s2 =
piecewise([1 <= x, Inf], [abs(x) < 1, -1/(x - 1)])
```

## Taylor Series

The statements

```
syms x
f = 1/(5 + 4*cos(x));
T = taylor(f, 8)
```

return

```
T =
(49*x^6)/131220 + (5*x^4)/1458 + (2*x^2)/81 + 1/9
```

which is all the terms up to, but not including, order eight in the Taylor series for  $f(x)$ :

$$\sum_{n=0}^{\infty} (x-a)^n \frac{f^{(n)}(a)}{n!}.$$

Technically,  $T$  is a Maclaurin series, since its base point is  $a = 0$ .

The command

```
pretty(T)
```

prints  $T$  in a format resembling typeset mathematics:

$$\frac{49 x^6}{131220} + \frac{5 x^4}{1458} + \frac{2 x^2}{81} + \frac{1}{9}$$

These commands

```
syms x
g = exp(x*sin(x))
t = taylor(g, 12, 2);
```

generate the first 12 nonzero terms of the Taylor series for  $g$  about  $x = 2$ .

$t$  is a large expression; enter

```
size(char(t))

ans =
      1      109959
```

to find that  $t$  has more than 100,000 characters in its printed form. In order to proceed with using  $t$ , first simplify its presentation:

```
t = simplify(t);
size(char(t))

ans =
      1      11585
```

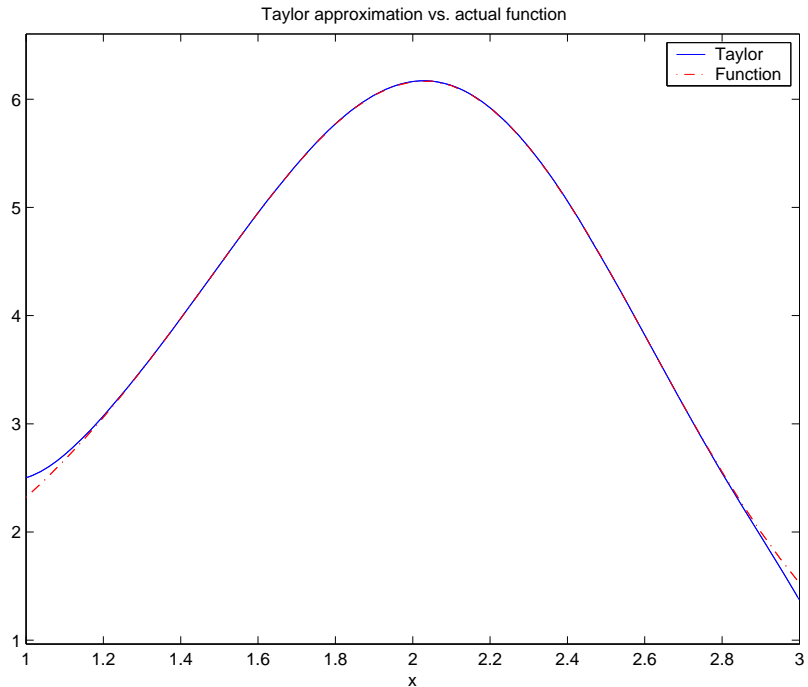
To simplify  $t$  even further, use the `simple` function:

```
t = simple(t);
size(char(t))

ans =
      1      6988
```

Next, plot these functions together to see how well this Taylor approximation compares to the actual function  $g$ :

```
xd = 1:0.05:3; yd = subs(g,x,xd);  
ezplot(t, [1, 3]); hold on;  
plot(xd, yd, 'r-.')  
title('Taylor approximation vs. actual function');  
legend('Taylor','Function')
```



Special thanks to Professor Gunnar Bäckström of UMEA in Sweden for this example.

## Calculus Example

This section describes how to analyze a simple function to find its asymptotes, maximum, minimum, and inflection point. The section covers the following topics:

- “Defining the Function” on page 3-23
- “Finding the Asymptotes” on page 3-24
- “Finding the Maximum and Minimum” on page 3-26
- “Finding the Inflection Point” on page 3-28

## Defining the Function

The function in this example is

$$f(x) = \frac{3x^2 + 6x - 1}{x^2 + x - 3}.$$

To create the function, enter the following commands:

```
syms x
num = 3*x^2 + 6*x - 1;
denom = x^2 + x - 3;
f = num/denom
```

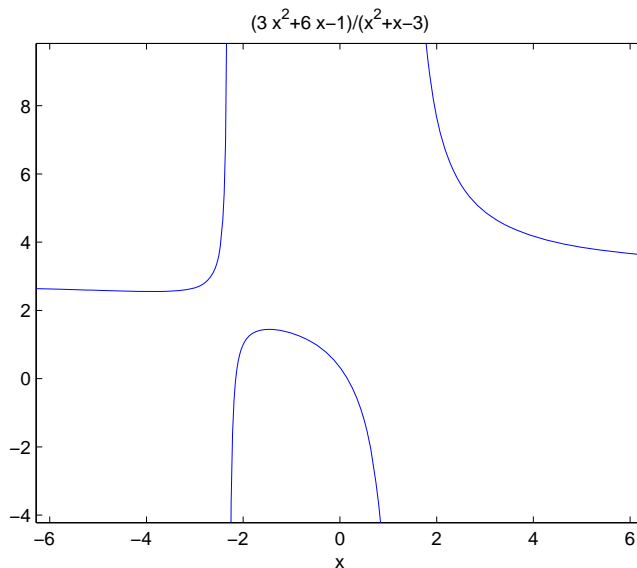
This returns

```
f =
(3*x^2 + 6*x - 1)/(x^2 + x - 3)
```

You can plot the graph of  $f$  by entering

```
ezplot(f)
```

This displays the following plot.



### Finding the Asymptotes

To find the horizontal asymptote of the graph of  $f$ , take the limit of  $f$  as  $x$  approaches positive infinity:

```
limit(f, inf)
ans =
3
```

The limit as  $x$  approaches negative infinity is also 3. This tells you that the line  $y = 3$  is a horizontal asymptote to the graph.

To find the vertical asymptotes of  $f$ , set the denominator equal to 0 and solve by entering the following command:

```
roots = solve(denom)
```

This returns to solutions to  $x^2 + x - 3 = 0$ :

```
roots =
```

$$13^{(1/2)}/2 - 1/2$$

$$- 13^{(1/2)}/2 - 1/2$$

This tells you that vertical asymptotes are the lines

$$x = \frac{-1 + \sqrt{13}}{2},$$

and

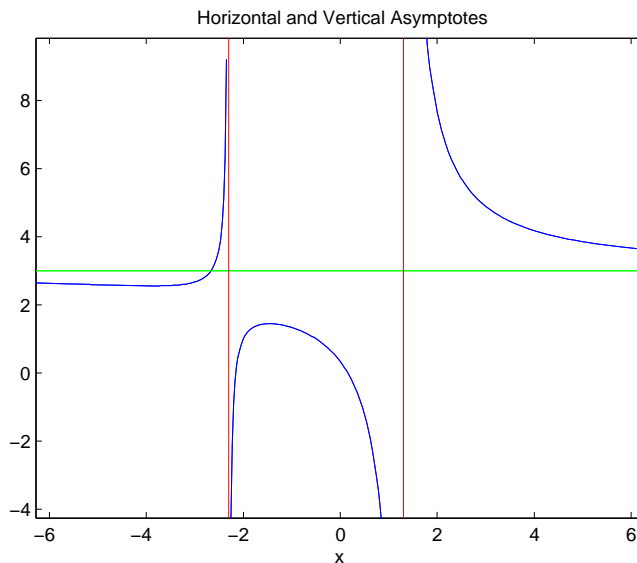
$$x = \frac{-1 - \sqrt{13}}{2}.$$

You can plot the horizontal and vertical asymptotes with the following commands:

```
ezplot(f)
hold on % Keep the graph of f in the figure
% Plot horizontal asymptote
plot([-2*pi 2*pi], [3 3], 'g')
% Plot vertical asymptotes
plot(double(roots(1))*[1 1], [-5 10], 'r')
plot(double(roots(2))*[1 1], [-5 10], 'r')
title('Horizontal and Vertical Asymptotes')
hold off
```

Note that roots must be converted to double to use the plot command.

The preceding commands display the following figure.



To recover the graph of  $f$  without the asymptotes, enter

```
ezplot(f)
```

### Finding the Maximum and Minimum

You can see from the graph that  $f$  has a local maximum somewhere between the points  $x = -2$  and  $x = 0$ , and might have a local minimum between  $x = -6$  and  $x = -2$ . To find the  $x$ -coordinates of the maximum and minimum, first take the derivative of  $f$ :

```
f1 = diff(f)
```

This returns

```
f1 =
(6*x + 6)/(x^2 + x - 3) - ((2*x + 1)*(3*x^2 + 6*x - 1))/(x^2 + x - 3)^2
```

To simplify this expression, enter

```
f1 = simplify(f1)
```



which returns

$$f1 = \\ -(3*x^2 + 16*x + 17)/(x^2 + x - 3)^2$$

You can display f1 in a more readable form by entering

```
pretty(f1)
```

which returns

$$\frac{3x^2 + 16x + 17}{(x^2 + x - 3)^2}$$

Next, set the derivative equal to 0 and solve for the critical points:

```
crit_pts = solve(f1)
```

This returns

$$\text{crit\_pts} = \\ \begin{aligned} &13^{(1/2)}/3 - 8/3 \\ &- 13^{(1/2)}/3 - 8/3 \end{aligned}$$

It is clear from the graph of  $f$  that it has a local minimum at

$$x_1 = \frac{-8 - \sqrt{13}}{3},$$

and a local maximum at

$$x_2 = \frac{-8 + \sqrt{13}}{3}.$$

---

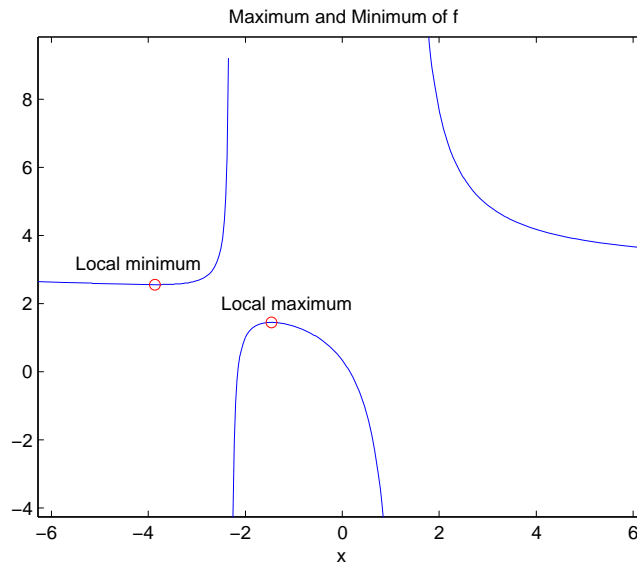
**Note** MATLAB does not always return the roots to an equation in the same order.

---

You can plot the maximum and minimum of  $f$  with the following commands:

```
ezplot(f)
hold on
plot(double(crit_pts), double(subs(f,crit_pts)), 'ro')
title('Maximum and Minimum of f')
text(-5.5,3.2,'Local minimum')
text(-2.5,2,'Local maximum')
hold off
```

This displays the following figure.



### Finding the Inflection Point

To find the inflection point of  $f$ , set the second derivative equal to 0 and solve.

```
f2 = diff(f1);
inflec_pt = solve(f2);
double(inflec_pt)
```

This returns

```
ans =
-5.2635
-1.3682 - 0.8511i
-1.3682 + 0.8511i
```

In this example, only the first entry is a real number, so this is the only inflection point. (Note that in other examples, the real solutions might not be the first entries of the answer.) Since you are only interested in the real solutions, you can discard the last two entries, which are complex numbers.

```
inflec_pt = inflec_pt(1)
```

To see the symbolic expression for the inflection point, enter

```
pretty(simplify(inflec_pt))
```

This returns

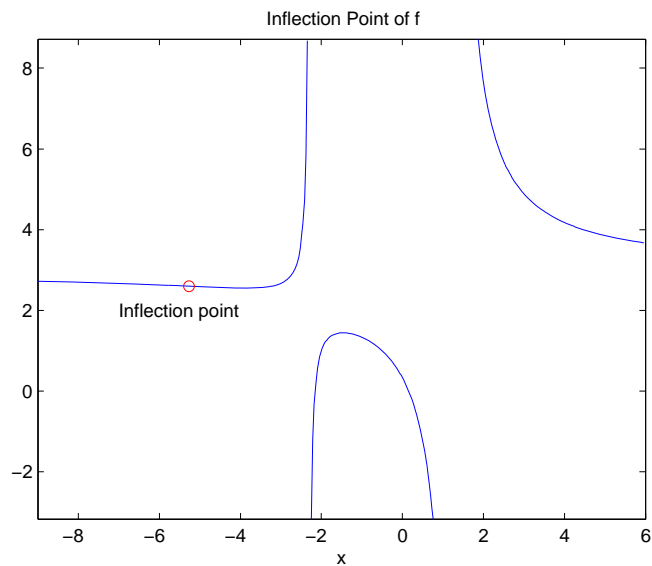
$$-\frac{13}{1} - \sqrt[3]{\frac{169}{54}} - \frac{2197^{1/2} \sqrt[3]{8}}{18 \sqrt[3]{3}}$$

To plot the inflection point, enter

```
ezplot(f, [-9 6])
hold on
```

```
plot(double(inflec_pt), double(subs(f,inflec_pt)),'ro')
title('Inflection Point of f')
text(-7,2,'Inflection point')
hold off
```

The extra argument, `[-9 6]`, in `ezplot` extends the range of  $x$  values in the plot so that you see the inflection point more clearly, as shown in the following figure.



## Extended Calculus Example

This section presents an extended example that illustrates how to find the maxima and minima of a function. The section covers the following topics:

- “Defining the Function” on page 3-31
- “Finding the Zeros of  $f_3$ ” on page 3-32
- “Finding the Maxima and Minima of  $f_2$ ” on page 3-36
- “Integrating” on page 3-37

## Defining the Function

The starting point for the example is the function

$$f(x) = \frac{1}{5 + 4 \cos(x)}.$$

You can create the function with the commands

```
syms x
f = 1/(5+4*cos(x))
```

which return

```
f =
1/(4*cos(x) + 5)
```

The example shows how to find the maximum and minimum of the second derivative of  $f(x)$ . To compute the second derivative, enter

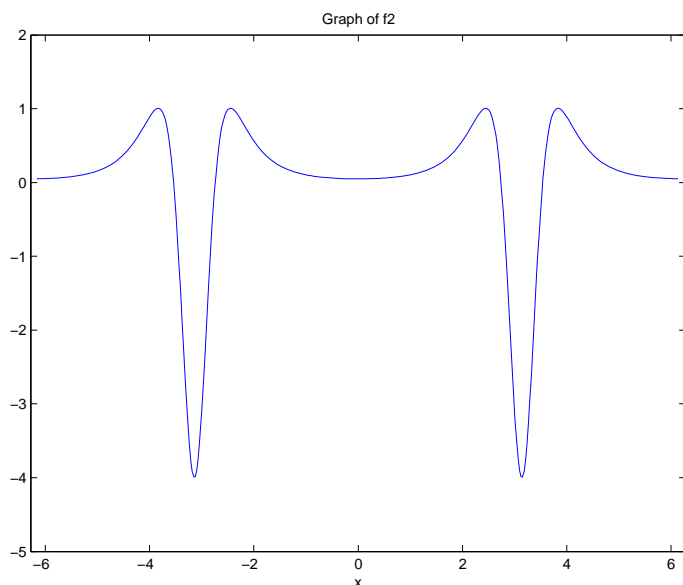
```
f2 = diff(f, 2)
```

which returns

```
f2 =
(4*cos(x))/(4*cos(x) + 5)^2 + (32*sin(x)^2)/(4*cos(x) + 5)^3
```

Equivalently, you can type `f2 = diff(f, x, 2)`. The default scaling in `ezplot` cuts off part of the graph of  $f2$ . You can set the axes limits manually to see the entire function:

```
ezplot(f2)
axis([-2*pi 2*pi -5 2])
title('Graph of f2')
```



From the graph, it appears that the maximum value of  $f''(x)$  is 1 and the minimum value is -4. As you will see, this is not quite true. To find the exact values of the maximum and minimum, you only need to find the maximum and minimum on the interval  $(-\pi, \pi]$ . This is true because  $f''(x)$  is periodic with period  $2\pi$ , so that the maxima and minima are simply repeated in each translation of this interval by an integer multiple of  $2\pi$ . The next two sections explain how to do find the maxima and minima.

### Finding the Zeros of f3

The maxima and minima of  $f''(x)$  occur at the zeros of  $f'''(x)$ . The statements

```
f3 = diff(f2);  
pretty(f3)
```

compute  $f'''(x)$  and display it in a more readable form:

$$\frac{384 \sin^3(x)}{(4 \cos(x) + 5)^4} - \frac{4 \sin(x)}{(4 \cos(x) + 5)^2} + \frac{96 \cos(x) \sin(x)}{(4 \cos(x) + 5)^3}$$

You can simplify this expression using the statements

```
f3 = simple(f3);
pretty(f3)
```

$$\frac{4 \sin(x) (-16 \cos^2(x) + 80 \cos(x) + 71)}{(4 \cos(x) + 5)^4}$$

Now, to find the zeros of  $f'''(x)$ , enter

```
zeros = solve(f3)
```

This returns a 5-by-1 symbolic matrix

```
zeros =
acos(5/2 - (3*19^(1/2))/4)
acos((3*19^(1/2))/4 + 5/2)
0
-acos(5/2 - 3/4*19^(1/2))
-acos(3/4*19^(1/2) + 5/2)
```

each of whose entries is a zero of  $f'''(x)$ . The commands

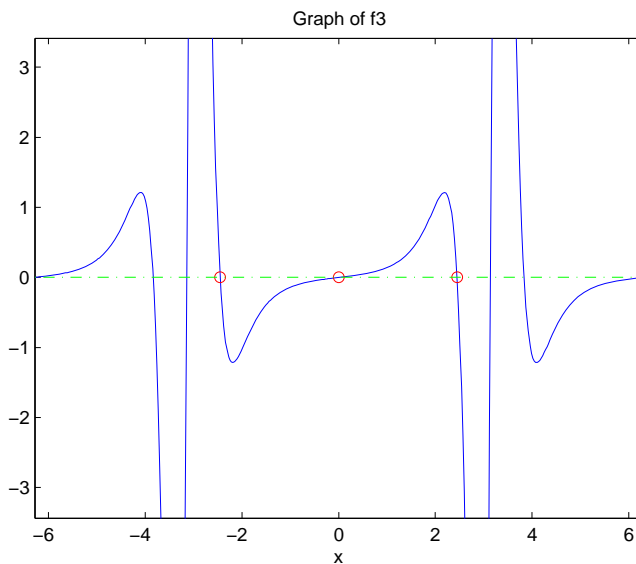
```
format;
% Default format of 5 digits
zerosd = double(zeros)
```

convert the zeros to double form:

```
zerosd =
    2.4483
         0 + 2.4381i
         0
    -2.4483
         0 - 2.4381i
```

So far, you have found three real zeros and two complex zeros. However, as the following graph of  $f_3$  shows, these are not all its zeros:

```
ezplot(f3)
hold on;
plot(zerosd,0*zerosd,'ro') % Plot zeros
plot([-2*pi,2*pi], [0,0], 'g-.'); % Plot x-axis
title('Graph of f3')
```



The red circles in the graph correspond to `zerosd(1)`, `zerosd(3)`, and `zerosd(4)`. As you can see in the graph, there are also zeros at  $\pm\pi$ . The additional zeros occur because  $f'''(x)$  contains a factor of  $\sin(x)$ , which is



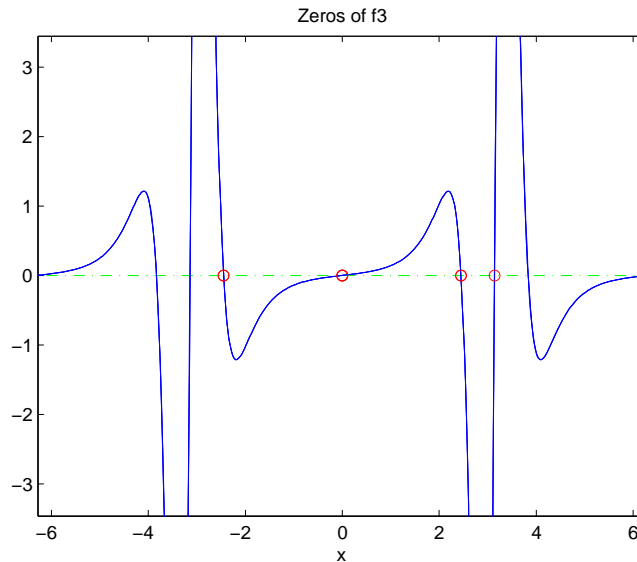
zero at integer multiples of  $\pi$ . The function, `solve(sin(x))`, however, only finds the zero at  $x = 0$ .

A complete list of the zeros of  $f'''(x)$  in the interval  $(-\pi, \pi]$  is

```
zerosd = [zerosd(1) zerosd(3) zerosd(4) pi];
```

You can display these zeros on the graph of  $f'''(x)$  with the following commands:

```
ezplot(f3)
hold on;
plot(zerosd,0*zerosd,'ro')
plot([-2*pi,2*pi], [0,0],'g-.'); % Plot x-axis
title('Zeros of f3')
hold off;
```



## Finding the Maxima and Minima of $f_2$

To find the maxima and minima of  $f''(x)$ , calculate the value of  $f''(x)$  at each of the zeros of  $f'''(x)$ . To do so, substitute zeros into  $f_2$  and display the result below zeros:

```
[zerosd; subs(f2,zerosd)]

ans =
    2.4483         0   -2.4483    3.1416
    1.0051    0.0494    1.0051   -4.0000
```

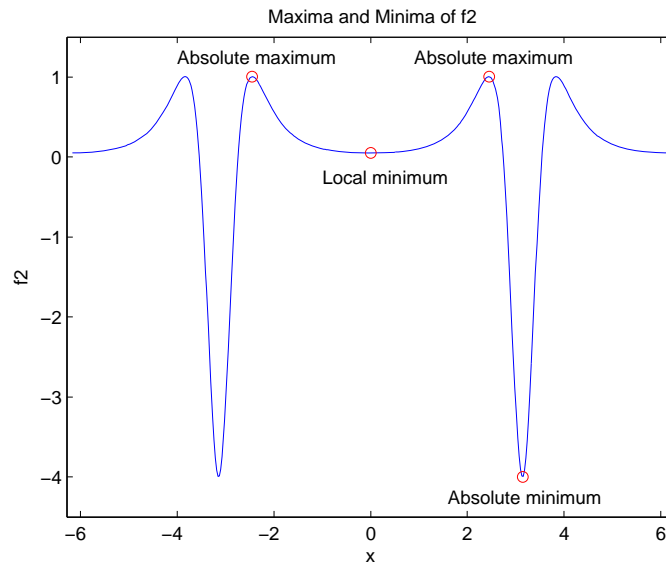
This shows the following:

- $f''(x)$  has an absolute maximum at  $x = \pm 2.4483$ , whose value is 1.0051.
- $f''(x)$  has an absolute minimum at  $x = \pi$ , whose value is -4.
- $f''(x)$  has a local minimum at  $x = 0$ , whose value is 0.0494.

You can display the maxima and minima with the following commands:

```
clf
ezplot(f2)
axis([-2*pi 2*pi -4.5 1.5])
ylabel('f2');
title('Maxima and Minima of f2')
hold on
plot(zerosd, subs(f2,zerosd), 'ro')
text(-4, 1.25, 'Absolute maximum')
text(-1,-0.25,'Local minimum')
text(.9, 1.25, 'Absolute maximum')
text(1.6, -4.25, 'Absolute minimum')
hold off;
```

This displays the following figure.



The preceding analysis shows that the actual range of  $f''(x)$  is  $[-4, 1.0051]$ .

## Integrating

Integrate  $f(x)$ :

$$F = \text{int}(f)$$

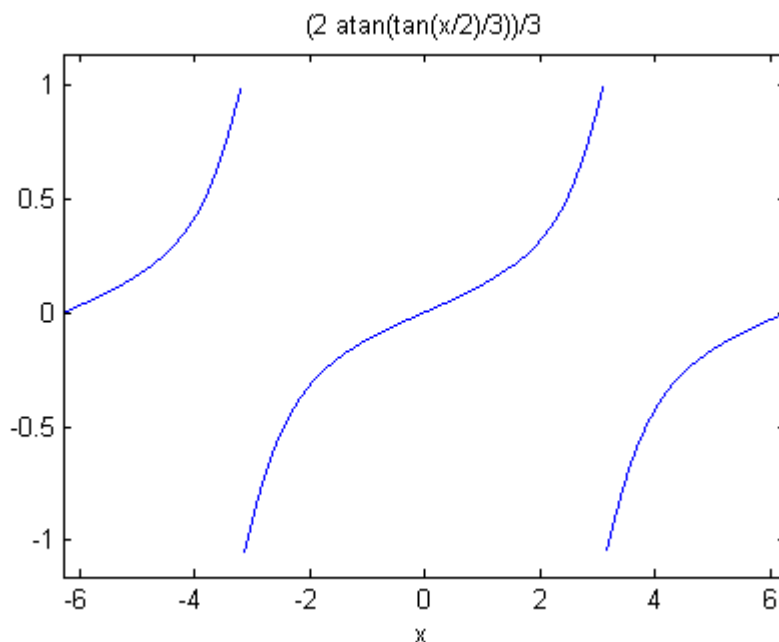
The result

$$F = (2 * \text{atan}(\tan(x/2)/3))/3$$

involves the arctangent function.

Note that  $F(x)$  is not an antiderivative of  $f(x)$  for all real numbers, since it is discontinuous at odd multiples of  $\pi$ , where  $\tan(x)$  is singular. You can see the gaps in  $F(x)$  in the following figure.

$$\text{ezplot}(F)$$



To change  $F(x)$  into a true antiderivative of  $f(x)$  that is differentiable everywhere, you can add a step function to  $F(x)$ . The height of the steps is the height of the gaps in the graph of  $F(x)$ . You can determine the height of the gaps by taking the limits of  $F(x)$  as  $x$  approaches  $\pi$  from the left and from the right. The limit from the left is

```
limit(F, x, pi, 'left')
```

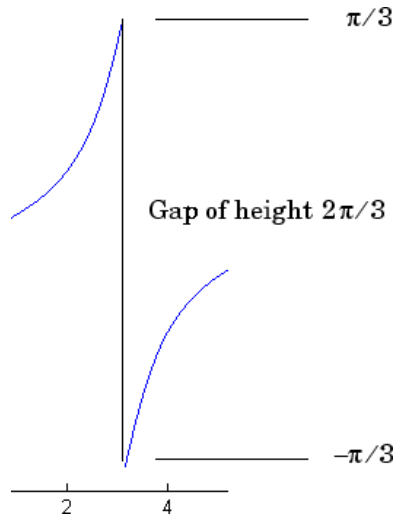
```
ans =  
pi/3
```

On the other hand, the limit from the right is

```
limit(F, x, pi, 'right')
```

```
ans =  
-pi/3
```

The height of the gap is the distance between the left- and right-hand limits, which is  $2\pi/3$  as shown in the following figure.

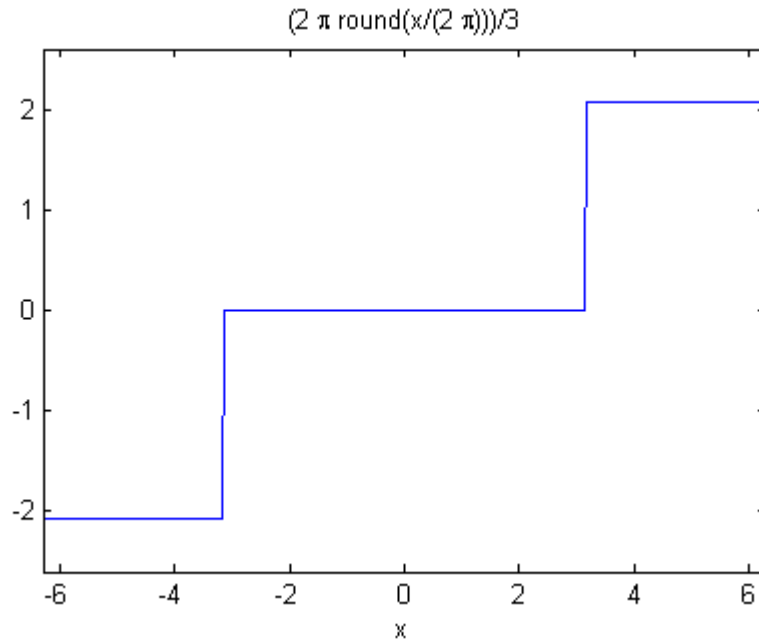


You can create the step function using the `round` function, which rounds numbers to the nearest integer, as follows:

```
J = sym(2*pi/3)*sym('round(x/(2*pi))');
```

Each step has width  $2\pi$ , and the jump from one step to the next is  $2\pi/3$ , as shown in the following figure, generated with

```
ezplot(J, [-2*pi 2*pi])
```

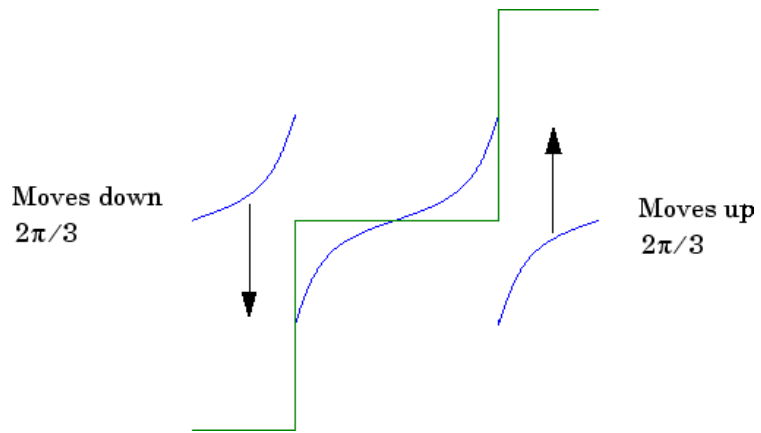


Next, add the step function  $J(x)$  to  $F(x)$  with the following code:

```
F1 = F + J
```

```
F1 =  
(2*atan(tan(x/2)/3))/3 + (2*pi*round(x/(2*pi)))/3
```

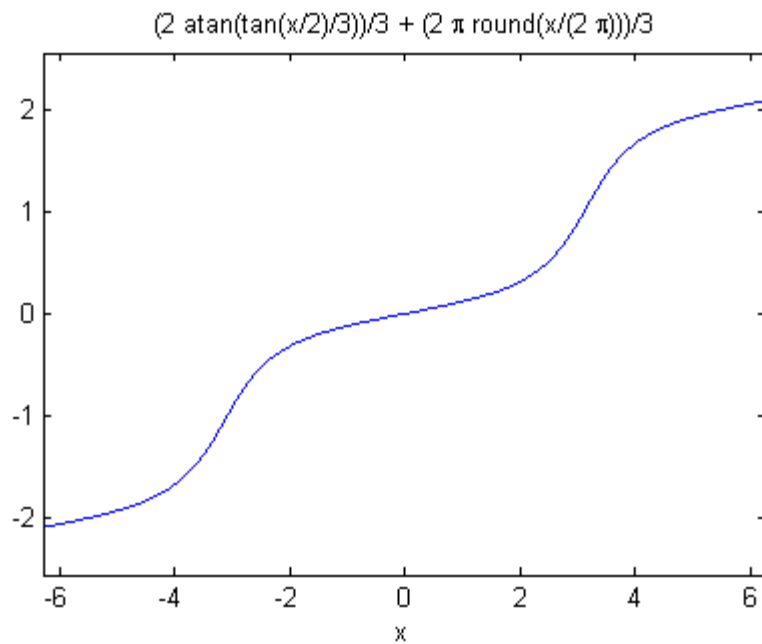
Adding the step function raises the section of the graph of  $F(x)$  on the interval  $[\pi, 3\pi)$  up by  $2\pi/3$ , lowers the section on the interval  $(-3\pi, -\pi]$  down by  $2\pi/3$ , and so on, as shown in the following figure.



When you plot the result by entering

`ezplot(F1)`

you see that this representation does have a continuous graph.



## Simplifications and Substitutions

### In this section...

“Simplifications” on page 3-42

“Substitutions” on page 3-53

### Simplifications

Here are three different symbolic expressions.

```
syms x
f = x^3 - 6*x^2 + 11*x - 6;
g = (x - 1)*(x - 2)*(x - 3);
h = -6 + (11 + (-6 + x)*x)*x;
```

Here are their prettyprinted forms, generated by

```
pretty(f);
pretty(g);
pretty(h)
```

$$x^3 - 6x^2 + 11x - 6$$

$$(x - 1)(x - 2)(x - 3)$$

$$x(x(x - 6) + 11) - 6$$

These expressions are three different representations of the same mathematical function, a cubic polynomial in  $x$ .

Each of the three forms is preferable to the others in different situations. The first form,  $f$ , is the most commonly used representation of a polynomial. It is simply a linear combination of the powers of  $x$ . The second form,  $g$ , is the factored form. It displays the roots of the polynomial and is the most accurate for numerical evaluation near the roots. But, if a polynomial does not have such simple roots, its factored form may not be so convenient. The third form,



$h$ , is the Horner, or nested, representation. For numerical evaluation, it involves the fewest arithmetic operations and is the most accurate for some other ranges of  $x$ .

The symbolic simplification problem involves the verification that these three expressions represent the same function. It also involves a less clearly defined objective — which of these representations is “the simplest”?

This toolbox provides several functions that apply various algebraic and trigonometric identities to transform one representation of a function into another, possibly simpler, representation. These functions are `collect`, `expand`, `horner`, `factor`, `simplify`, and `simple`.

### **collect**

The statement `collect(f)` views  $f$  as a polynomial in its symbolic variable, say  $x$ , and collects all the coefficients with the same power of  $x$ . A second argument can specify the variable in which to collect terms if there is more than one candidate. Here are a few examples.

<b>f</b>	<b>collect(f)</b>
<pre>syms x; f = (x-1)*(x-2)*(x-3);</pre>	<pre>collect(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms x; f = x*(x*(x - 6) + 11) - 6;</pre>	<pre>collect(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms x t; f = (1+x)*t + x*t;</pre>	<pre>collect(f)  ans = (2*t)*x + t</pre>

**expand**

The statement `expand(f)` distributes products over sums and applies other identities involving functions of sums as shown in the examples below.

<b>f</b>	<b>expand(f)</b>
<pre>syms a x y; f = a*(x + y);</pre>	<pre>expand(f)  ans = a*x + a*y</pre>
<pre>syms x; f = (x - 1)*(x - 2)*(x - 3);</pre>	<pre>expand(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms x; f = x*(x*(x - 6) + 11) - 6;</pre>	<pre>expand(f)  ans = x^3 - 6*x^2 + 11*x - 6</pre>
<pre>syms a b; f = exp(a + b);</pre>	<pre>expand(f)  ans = exp(a)*exp(b)</pre>
<pre>syms x y; f = cos(x + y);</pre>	<pre>expand(f)  ans = cos(x)*cos(y) - sin(x)*sin(y)</pre>

<b>f</b>	<b>expand(f)</b>
<pre>syms x; f = cos(3*acos(x));</pre>	<pre>expand(f) ans = 3*x*(x^2 - 1) + x^3</pre>
<pre>syms x; f = 3*x*(x^2 - 1) + x^3;</pre>	<pre>expand(f) ans = 4*x^3 - 3*x</pre>

### horner

The statement `horner(f)` transforms a symbolic polynomial `f` into its Horner, or nested, representation as shown in the following examples.

<b>f</b>	<b>horner(f)</b>
<pre>syms x; f = x^3 - 6*x^2 + 11*x - 6;</pre>	<pre>horner(f) ans = x*(x*(x - 6) + 11) - 6</pre>
<pre>syms x; f = 1.1 + 2.2*x + 3.3*x^2;</pre>	<pre>horner(f) ans = x*((33*x)/10 + 11/5) + 11/10</pre>

### factor

If `f` is a polynomial with rational coefficients, the statement

```
factor(f)
```

expresses `f` as a product of polynomials of lower degree with rational coefficients. If `f` cannot be factored over the rational numbers, the result is `f` itself. Here are several examples.

f	factor(f)
<pre>syms x; f = x^3 - 6*x^2 + 11*x - 6;</pre>	<pre>factor(f)  ans = (x - 3)*(x - 1)*(x - 2)</pre>
<pre>syms x; f = x^3 - 6*x^2 + 11*x - 5;</pre>	<pre>factor(f)  ans = x^3 - 6*x^2 + 11*x - 5</pre>
<pre>syms x; f = x^6 + 1;</pre>	<pre>factor(f)  ans = (x^2 + 1)*(x^4 - x^2 + 1)</pre>

Here is another example involving `factor`. It factors polynomials of the form  $x^n + 1$ . This code

```
syms x;
n = (1:9)';
p = x.^n + 1;
f = factor(p);
[p, f]
```

returns a matrix with the polynomials in its first column and their factored forms in its second.

```
ans =
[ x + 1, x + 1]
[ x^2 + 1, x^2 + 1]
[ x^3 + 1, (x + 1)*(x^2 - x + 1)]
[ x^4 + 1, x^4 + 1]
[ x^5 + 1, (x + 1)*(x^4 - x^3 + x^2 - x + 1)]
[ x^6 + 1, (x^2 + 1)*(x^4 - x^2 + 1)]
[ x^7 + 1, (x + 1)*(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)]
[ x^8 + 1, x^8 + 1]
[ x^9 + 1, (x + 1)*(x^2 - x + 1)*(x^6 - x^3 + 1)]
```

As an aside at this point, `factor` can also factor symbolic objects containing integers. This is an alternative to using the `factor` function in the MATLAB `specfun` directory. For example, the following code segment

```
N = sym(1);
for k = 2:11
    N(k) = 10*N(k-1)+1;
end
[N' factor(N')]
```

displays the factors of symbolic integers consisting of 1s:

```
ans =
[          1,          1]
[          11,         11]
[          111,        3*37]
[          1111,       11*101]
[          11111,      41*271]
[          111111,   3*7*11*13*37]
[          1111111,  239*4649]
[          11111111, 11*73*101*137]
[          111111111, 3^2*37*333667]
[          1111111111, 11*41*271*9091]
[          11111111111, 21649*513239]
```

## **simplify**

The `simplify` function is a powerful, general purpose tool that applies a number of algebraic identities involving sums, integral powers, square roots and other fractional powers, as well as a number of functional identities involving trig functions, exponential and log functions, Bessel functions, hypergeometric functions, and the gamma function. Here are some examples.

<b>f</b>	<b>simplify(f)</b>
<pre>syms x; f = x*(x*(x - 6) + 11) - 6;</pre>	<pre>simplify(f)  ans = (x - 1)*(x - 2)*(x - 3)</pre>

<b>f</b>	<b>simplify(f)</b>
<pre>syms x; f = (1 - x^2)/(1 - x);</pre>	<pre>simplify(f)  ans = x + 1</pre>
<pre>syms a; f = (1/a^3 + 6/a^2 + 12/a + 8)^(1/3);</pre>	<pre>simplify(f)  ans = ((2*a + 1)^3/a^3)^(1/3)</pre>
<pre>syms x y; f = exp(x) * exp(y);</pre>	<pre>simplify(f)  ans = exp(x + y)</pre>
<pre>syms x; f = besselj(2, x) + besselj(0, x);</pre>	<pre>simplify(f)  ans = (2*besselj(1, x))/x</pre>
<pre>syms x; f = gamma(x + 1) - x*gamma(x);</pre>	<pre>simplify(f)  ans = 0</pre>
<pre>syms x; f = cos(x)^2 + sin(x)^2;</pre>	<pre>simplify(f)  ans = 1</pre>

You can also use the syntax `simplify(f, n)` where `n` is a positive integer that controls how many steps `simplify` takes. The default, when you don't provide an argument `n`, is 100 steps. For example,

```
z = diff(x/cos(x), 3)
```

```

z =
3/cos(x) + (6*sin(x)^2)/cos(x)^3 +...
(6*x*sin(x)^3)/cos(x)^4 + (5*x*sin(x))/cos(x)^2

simplify(z)

ans =
(- 3*cos(x)^3 - x*sin(x)*cos(x)^2 + 6*cos(x) +
6*x*sin(x))/cos(x)^4

simplify(z, 200)

ans =
(6*cos(x) - 3*cos(x)^3 + sin(x)*(6*x-x*cos(x)^2))/cos(x)^4

```

## simple

The `simple` function has the unorthodox mathematical goal of finding a simplification of an expression that has the fewest number of characters. Of course, there is little mathematical justification for claiming that one expression is “simpler” than another just because its ASCII representation is shorter, but this often proves satisfactory in practice.

The `simple` function achieves its goal by independently applying `simplify`, `collect`, `factor`, and other simplification functions to an expression and keeping track of the lengths of the results. The `simple` function then returns the shortest result.

The `simple` function has several forms, each returning different output. The form `simple(f)` displays each trial simplification and the simplification function that produced it in the MATLAB command window. The `simple` function then returns the shortest result. For example, the command

```

syms x;
simple(cos(x)^2 + sin(x)^2)

```

displays the following alternative simplifications in the MATLAB command window along with the result:

```

simplify:
1

```

```
radsimp:  
cos(x)^2 + sin(x)^2  
  
simplify(100):  
1  
  
combine(sincos):  
1  
  
combine(sinhcosh):  
cos(x)^2 + sin(x)^2  
  
combine(ln):  
cos(x)^2 + sin(x)^2  
  
factor:  
cos(x)^2 + sin(x)^2  
  
expand:  
cos(x)^2 + sin(x)^2  
  
combine:  
cos(x)^2 + sin(x)^2  
  
rewrite(exp):  
((i*exp(i*x))/2 - i/(2*exp(i*x)))^2 + (exp(i*x)/2 + 1/(2*exp(i*x)))^2  
  
rewrite(sincos):  
cos(x)^2 + sin(x)^2  
  
rewrite(sinhcosh):  
cosh(-i*x)^2 - sinh(-i*x)^2  
  
rewrite(tan):  
(tan(x/2)^2 - 1)^2/(tan(x/2)^2 + 1)^2 + (4*tan(x/2)^2)/(tan(x/2)^2 + 1)^2  
  
mwcos2sin:  
1
```



```
collect(x):  
cos(x)^2 + sin(x)^2  
  
ans =  
1
```

This form is useful when you want to check, for example, whether the shortest form is indeed the simplest. If you are not interested in how `simple` achieves its result, use the form `f = simple(f)`. This form simply returns the shortest expression found. For example, the statement

```
f = simple(cos(x)^2 + sin(x)^2)
```

returns

```
f =  
1
```

If you want to know which simplification returned the shortest result, use the multiple output form `[f, how] = simple(f)`. This form returns the shortest result in the first variable and the simplification method used to achieve the result in the second variable. For example, the statement

```
[f, how] = simple(cos(x)^2 + sin(x)^2)
```

returns

```
f =  
1  
  
how =  
simplify
```

The `simple` function sometimes improves on the result returned by `simplify`, one of the simplifications that it tries. For example, when applied to the examples given for `simplify`, `simple` returns a simpler (or at least shorter) result as shown:

<b>f</b>	<b>simplify(f)</b>	<b>simple(f)</b>
<pre>syms a positive; f = (1/a^3 + 6/a^2 + 12/a + 8)^(1/3);</pre>	<pre>simplify(f)  ans = ((2*a + 1)^3/a^3)^(1/3)</pre>	<pre>g = simple(f)  g = 1/a + 2</pre>
<pre>syms x; f = cos(x) + i*sin(x);</pre>	<pre>simplify(f)  ans = cos(x) + i*sin(x)</pre>	<pre>g = simple(f)  g = exp(i*x)</pre>

In some cases, it is advantageous to apply `simple` twice to obtain the effect of two different simplification functions. For example:

```
z = diff(x/cos(x),3)

z =
3/cos(x) + (6*sin(x)^2)/cos(x)^3 + (6*x*sin(x)^3)/cos(x)^4 + (5*x*sin(x))/cos(x)^2

z1 = simple(z)

z1 =
(6*cos(x) - 3*cos(x)^3 + sin(x)*(6*x -
x*cos(x)^2))/(sin(x)^2 - 1)^2

z2 = simple(simple(z))

z2 =
(6*cos(x) - 3*cos(x)^3 + sin(x)*(6*x - x*cos(x)^2))/cos(x)^4
```

The `simple` function is particularly effective on expressions involving trigonometric functions:

<b>f</b>	<b>simple(f)</b>
<pre>syms x; f = cos(x)^2 + sin(x)^2;</pre>	<pre>f = simple(f)  f = 1</pre>
<pre>syms x; f = 2*cos(x)^2 - sin(x)^2;</pre>	<pre>f = simple(f)  f = 3*cos(x)^2 - 1</pre>
<pre>syms x; f = cos(x)^2 - sin(x)^2;</pre>	<pre>f = simple(f)  f = cos(2*x)</pre>
<pre>syms x; f = cos(x) + i*sin(x);</pre>	<pre>f = simple(f)  f = exp(i*x)</pre>
<pre>syms x; f = cos(3*acos(x));</pre>	<pre>f = simple(f)  f = 4*x^3 - 3*x</pre>

## Substitutions

There are two functions for symbolic substitution: `subexpr` and `subs`.

### subexpr

These commands

```
syms a x
s = solve(x^3 + a*x + 1)
```

solve the equation  $x^3 + a*x + 1 = 0$  for the variable  $x$ :

```
s =
((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3) - a/(3*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3))

a/(6*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)) - ((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)/2...
- (3^(1/2)*i*(a/(3*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)))...
+ ((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3))/2

a/(6*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)) - ((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)/2...
+ (3^(1/2)*i*(a/(3*((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3)))...
+ ((a^3/27 + 1/4)^(1/2) - 1/2)^(1/3))/2
```

This long expression has many repeated pieces, or subexpressions. The `subexpr` function allows you to save these common subexpressions as well as the symbolic object rewritten in terms of the subexpressions. The subexpressions are saved in a column vector called `sigma`.

Continuing with the example

```
r = subexpr(s)
```

returns

```
sigma =
(a^3/27 + 1/4)^(1/2) - 1/2

r =
sigma^(1/3) - a/(3*sigma^(1/3))
a/(6*sigma^(1/3)) - sigma^(1/3)/2 - (3^(1/2)*i*(a/(3*sigma^(1/3)) + sigma^(1/3)))/2
a/(6*sigma^(1/3)) - sigma^(1/3)/2 + (3^(1/2)*i*(a/(3*sigma^(1/3)) + sigma^(1/3)))/2
```

Notice that `subexpr` creates the variable `sigma` in the MATLAB workspace. You can verify this by typing `whos`, or the command

```
sigma
```

which returns

```
sigma =
(a^3/27 + 1/4)^(1/2) - 1/2
```

**subs**

The following code finds the eigenvalues and eigenvectors of a circulant matrix A:

```

syms a b c
A = [a b c; b c a; c a b];
[v,E] = eig(A)

v =
[ (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (a - b)/(a - c), ...
  - (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (a - b)/(a - c), ...
  1]
[ - (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (b - c)/(a - c), ...
  (a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)/(a - c) - (b - c)/(a - c), ...
  1]
[ 1, 1, 1]

E =
[ -(a^2-a*b-a*c+b^2-b*c+c^2)^(1/2), 0, 0]
[ 0, (a^2-a*b-a*c+b^2-b*c+c^2)^(1/2), 0]
[ 0, 0, a+b+c]

```

---

**Note** MATLAB might return the eigenvalues that appear on the diagonal of E in a different order. In this case, the corresponding eigenvectors, which are the columns of v, will also appear in a different order.

---

Suppose you want to replace the rather lengthy expression  $(a^2 - a*b - a*c + b^2 - b*c + c^2)^{(1/2)}$  throughout v and E. First, use `subexpr`:

```
E = subexpr(E, 'S')
```

which returns

```

S =
(a^2 - a*b - a*c + b^2 - b*c + c^2)^(1/2)

E =

```

```
[ -S, 0,      0]
[  0, S,      0]
[  0, 0, a + b + c]
```

Next, substitute the symbol S into v with

```
v = simplify(subs(v, S, 'S'))

v =
[ (S - a + b)/(a - c), -(S + a - b)/(a - c), 1]
[ -(S + b - c)/(a - c), (S - b + c)/(a - c), 1]
[                      1,                      1, 1]
```

Now suppose you want to evaluate v at a = 10. Use the subs command:

```
subs(v, a, 10)
```

This replaces all occurrences of a in v with 10:

```
ans =
[ -(S + b - 10)/(c - 10), (S - b + 10)/(c - 10), 1]
[ (S + b - c)/(c - 10), -(S - b + c)/(c - 10), 1]
[                      1,                      1, 1]
```

Notice, however, that the symbolic expression that S represents is unaffected by this substitution. That is, the symbol a in S is not replaced by 10. The subs command is also a useful function for substituting in a variety of values for several variables in a particular expression. For example, suppose that in addition to substituting a = 10 in S, you also want to substitute the values for 2 and 10 for b and c, respectively. The way to do this is to set values for a, b, and c in the workspace. Then subs evaluates its input using the existing symbolic and double variables in the current workspace. In the example, you first set

```
a = 10; b = 2; c = 10;
subs(S)
```

```
ans =
      8
```

To look at the contents of the workspace, type:

whos

which gives

Name	Size	Bytes	Class	Attributes
A	3x3	622	sym	
E	3x3	1144	sym	
S	1x1	184	sym	
a	1x1	8	double	
ans	1x1	8	double	
b	1x1	8	double	
c	1x1	8	double	
v	3x3	1144	sym	

a, b, and c are now variables of class `double` while A, E, S, and v remain symbolic expressions (class `sym`).

If you want to preserve a, b, and c as symbolic variables, but still alter their value within S, use this procedure.

```
syms a b c
subs(S, {a, b, c}, {10, 2, 10})

ans =
     8
```

Typing `whos` reveals that a, b, and c remain 1-by-1 `sym` objects.

The `subs` command can be combined with `double` to evaluate a symbolic expression numerically. Suppose you have the following expressions

```
syms t
M = (1 - t^2)*exp(-1/2*t^2);
P = (1 - t^2)*sech(t);
```

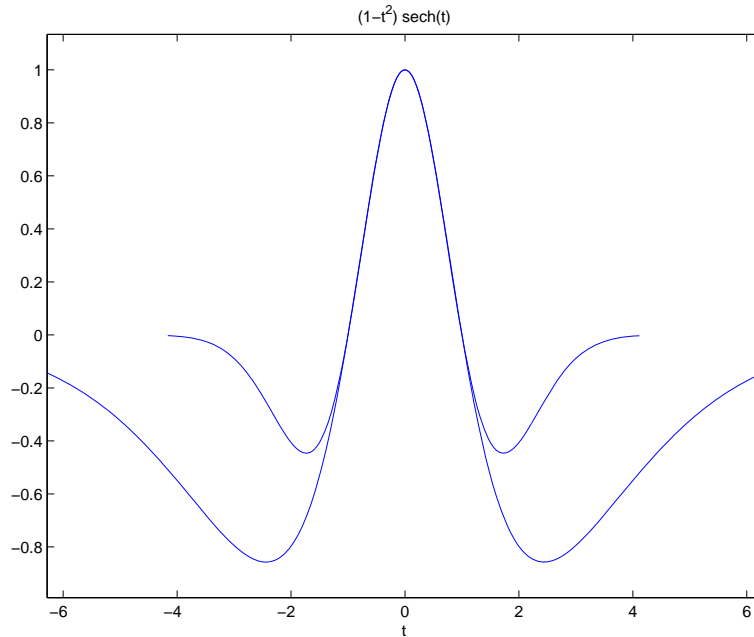
and want to see how M and P differ graphically.

One approach is to type

```
ezplot(M);
hold on;
```

```
ezplot(P);  
hold off;
```

but this plot does not readily help you identify the curves.

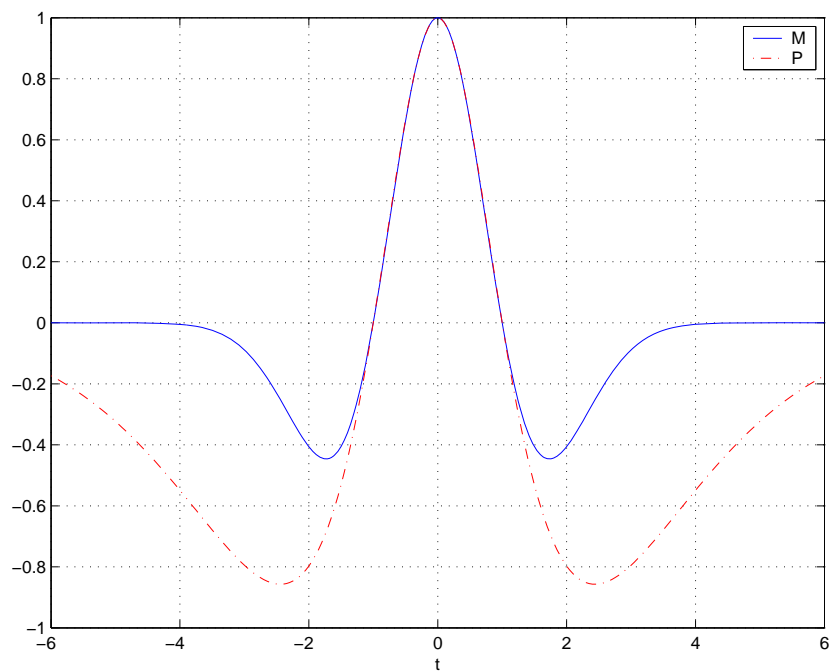


Instead, combine `subs`, `double`, and `plot`:

```
T = -6:0.05:6;  
MT = double(subs(M, t, T));  
PT = double(subs(P, t, T));  
plot(T, MT, 'b', T, PT, 'r-.');  
title(' ');  
legend('M','P');  
xlabel('t'); grid;
```

to produce a multicolored graph that indicates the difference between M and P.





Finally the use of subs with strings greatly facilitates the solution of problems involving the Fourier, Laplace, or  $z$ -transforms. See the section “Integral Transforms and Z-Transforms” on page 3-102 for complete details.

## Variable-Precision Arithmetic

In this section...
“Overview” on page 3-60
“Example: Using the Different Kinds of Arithmetic” on page 3-61
“Another Example Using Different Kinds of Arithmetic” on page 3-64

### Overview

There are three different kinds of arithmetic operations in this toolbox:

Numeric	MATLAB floating-point arithmetic
Rational	MuPAD software exact symbolic arithmetic
VPA	MuPAD software variable-precision arithmetic

For example, the MATLAB statements

```
format long
1/2 + 1/3
```

use numeric computation to produce

```
ans =
0.8333333333333333
```

With Symbolic Math Toolbox software, the statement

```
sym(1/2) + 1/3
```

uses symbolic computation to yield

```
ans =
5/6
```

And, also with the toolbox, the statements

```
digits(25)
```

```
vpa('1/2 + 1/3')
```

use variable-precision arithmetic to return

```
ans =  
0.83333333333333333333333333333333
```

The floating-point operations used by numeric arithmetic are the fastest of the three, and require the least computer memory, but the results are not exact. The number of digits in the printed output of MATLAB double quantities is controlled by the `format` statement, but the internal representation is always the eight-byte floating-point representation provided by the particular computer hardware.

In the computation of the numeric result above, there are actually three roundoff errors, one in the division of 1 by 3, one in the addition of 1/2 to the result of the division, and one in the binary to decimal conversion for the printed output. On computers that use IEEE<sup>®</sup> floating-point standard arithmetic, the resulting internal value is the binary expansion of 5/6, truncated to 53 bits. This is approximately 16 decimal digits. But, in this particular case, the printed output shows only 15 digits.

The symbolic operations used by rational arithmetic are potentially the most expensive of the three, in terms of both computer time and memory. The results are exact, as long as enough time and memory are available to complete the computations.

Variable-precision arithmetic falls in between the other two in terms of both cost and accuracy. A global parameter, set by the function `digits`, controls the number of significant decimal digits. Increasing the number of digits increases the accuracy, but also increases both the time and memory requirements. The default value of `digits` is 32, corresponding roughly to floating-point accuracy.

## Example: Using the Different Kinds of Arithmetic

### Rational Arithmetic

By default, Symbolic Math Toolbox software uses rational arithmetic operations, i.e., MuPAD software's exact symbolic arithmetic. Rational

arithmetic is invoked when you create symbolic variables using the `sym` function.

The `sym` function converts a double matrix to its symbolic form. For example, if the double matrix is

```
format short;
A = [1.1,1.2,1.3;2.1,2.2,2.3;3.1,3.2,3.3]

A =
    1.1000    1.2000    1.3000
    2.1000    2.2000    2.3000
    3.1000    3.2000    3.3000
```

its symbolic form is:

```
S = sym(A)

S =
 [ 11/10,   6/5, 13/10]
 [ 21/10, 11/5, 23/10]
 [ 31/10, 16/5, 33/10]
```

For this matrix `A`, it is possible to discover that the elements are the ratios of small integers, so the symbolic representation is formed from those integers. On the other hand, the statement

```
E = [exp(1) (1 + sqrt(5))/2; log(3) rand]
```

returns a matrix

```
E =
    2.7183    1.6180
    1.0986    0.6324
```

whose elements are not the ratios of small integers, so

```
sym(E)
```

reproduces the floating-point representation in a symbolic form:

```
ans =
```

```
[ 3060513257434037/1125899906842624, 910872158600853/562949953421312]
[ 2473854946935173/2251799813685248, 1423946432832521/2251799813685248]
```

## Variable-Precision Numbers

Variable-precision numbers are distinguished from the exact rational representation by the presence of a decimal point. A power of 10 scale factor, denoted by 'e', is allowed. To use variable-precision instead of rational arithmetic, create your variables using the `vpa` function.

For matrices with purely double entries, the `vpa` function generates the representation that is used with variable-precision arithmetic. For example, if you apply `vpa` to the matrix `S` defined in the preceding section, with `digits(4)`, by entering

```
vpa(S)
```

MATLAB returns the output

```
ans =
[ 1.1, 1.2, 1.3]
[ 2.1, 2.2, 2.3]
[ 3.1, 3.2, 3.3]
```

Applying `vpa` to the matrix `E` defined in the preceding section, with `digits(25)`, by entering

```
digits(25)
F = vpa(E)
```

returns

```
F =
[ 2.718281828459045534884808, 1.618033988749894902525739]
[ 1.098612288668109560063613, 0.6323592462254095103446616]
```

## Converting to Floating-Point

To convert a rational or variable-precision number to its MATLAB floating-point representation, use the `double` function.

In the example, both `double(sym(E))` and `double(vpa(E))` return `E`.

## Another Example Using Different Kinds of Arithmetic

The next example is perhaps more interesting. Start with the symbolic expression

```
f = sym('exp(pi*sqrt(163))')
```

The statement

```
format long;  
double(f)
```

produces the printed floating-point value

```
ans =  
2.625374126407687e+017
```

Using the second argument of `vpa` to specify the number of digits,

```
vpa(f,18)
```

returns

```
ans =  
262537412640768744.0
```

and, too,

```
vpa(f,25)
```

returns

```
ans =  
262537412640768744.0
```

You might suspect that `f` actually has an integer value. This suspicion is reinforced by the 30 digit value:

```
vpa(f,30)
```

```
ans =  
262537412640768743.999999999999
```

Finally, the 40-digit value:

```
vpa(f,40)
```

```
ans =
```

```
262537412640768743.999999999992500725972
```

shows that  $f$  is very close to, but not exactly equal to, an integer.

## Linear Algebra

### In this section...

“Basic Algebraic Operations” on page 3-66

“Linear Algebraic Operations” on page 3-67

“Eigenvalues” on page 3-72

“Jordan Canonical Form” on page 3-77

“Singular Value Decomposition” on page 3-79

“Eigenvalue Trajectories” on page 3-82

### Basic Algebraic Operations

Basic algebraic operations on symbolic objects are the same as operations on MATLAB objects of class `double`. This is illustrated in the following example.

The Givens transformation produces a plane rotation through the angle  $t$ . The statements

```
syms t;  
G = [cos(t) sin(t); -sin(t) cos(t)]
```

create this transformation matrix.

```
G =  
[ cos(t),  sin(t)]  
[ -sin(t),  cos(t)]
```

Applying the Givens transformation twice should simply be a rotation through twice the angle. The corresponding matrix can be computed by multiplying  $G$  by itself or by raising  $G$  to the second power. Both

```
A = G*G
```

and

```
A = G^2
```



produce

```
A =
[ cos(t)^2 - sin(t)^2,    2*cos(t)*sin(t)]
[ (-2)*cos(t)*sin(t), cos(t)^2 - sin(t)^2]
```

The `simple` function

```
A = simple(A)
```

uses a trigonometric identity to return the expected form by trying several different identities and picking the one that produces the shortest representation.

```
A =
[ cos(2*t), sin(2*t)]
[ -sin(2*t), cos(2*t)]
```

The Givens rotation is an orthogonal matrix, so its transpose is its inverse. Confirming this by

```
I = G.' *G
```

which produces

```
I =
[ cos(t)^2 + sin(t)^2,    0]
[ 0, cos(t)^2 + sin(t)^2]
```

and then

```
I = simple(I)
```

```
I =
[ 1, 0]
[ 0, 1]
```

## Linear Algebraic Operations

The following examples show how to do several basic linear algebraic operations using Symbolic Math Toolbox software.

The command

```
H = hilb(3)
```

generates the 3-by-3 Hilbert matrix. With `format short`, MATLAB prints

```
H =  
    1.0000    0.5000    0.3333  
    0.5000    0.3333    0.2500  
    0.3333    0.2500    0.2000
```

The computed elements of `H` are floating-point numbers that are the ratios of small integers. Indeed, `H` is a MATLAB array of class `double`. Converting `H` to a symbolic matrix

```
H = sym(H)
```

gives

```
H =  
[ 1, 1/2, 1/3]  
[ 1/2, 1/3, 1/4]  
[ 1/3, 1/4, 1/5]
```

This allows subsequent symbolic operations on `H` to produce results that correspond to the infinitely precise Hilbert matrix, `sym(hilb(3))`, not its floating-point approximation, `hilb(3)`. Therefore,

```
inv(H)
```

produces

```
ans =  
[ 9, -36, 30]  
[-36, 192, -180]  
[ 30, -180, 180]
```

and

```
det(H)
```

yields

```
ans =
```

1/2160

You can use the backslash operator to solve a system of simultaneous linear equations. For example, the commands

```
% Solve Hx = b
b = [1; 1; 1];
x = H\b
```

produce the solution

```
x =
    3
   -24
    30
```

All three of these results, the inverse, the determinant, and the solution to the linear system, are the exact results corresponding to the infinitely precise, rational, Hilbert matrix. On the other hand, using `digits(16)`, the command

```
digits(16);
V = vpa(hilb(3))
```

returns

```
V =
[ 1.0, 0.5, 0.3333333333333333]
[ 0.5, 0.3333333333333333, 0.25]
[ 0.3333333333333333, 0.25, 0.2]
```

The decimal points in the representation of the individual elements are the signal to use variable-precision arithmetic. The result of each arithmetic operation is rounded to 16 significant decimal digits. When inverting the matrix, these errors are magnified by the matrix condition number, which for `hilb(3)` is about 500. Consequently,

```
inv(V)
```

which returns

```
ans =
[ 9.000000000000061, -36.00000000000032, 30.0000000000003]
```

```
[ -36.00000000000032, 192.0000000000017, -180.0000000000015]
[ 30.00000000000003, -180.0000000000015, 180.0000000000014]
```

shows the loss of two digits. So does

```
1/det(V)
```

which gives

```
ans =
2160.000000000018
```

and

```
V\b
```

which is

```
ans =
3.000000000000041
-24.00000000000021
30.00000000000019
```

Since H is nonsingular, calculating the null space of H with the command

```
null(H)
```

returns an empty matrix:

```
ans =
[ empty sym ]
```

Calculating the column space of H with

```
colspace(H)
```

returns a permutation of the identity matrix:

```
ans =
[ 1, 0, 0]
[ 0, 1, 0]
[ 0, 0, 1]
```

A more interesting example, which the following code shows, is to find a value  $s$  for  $H(1,1)$  that makes  $H$  singular. The commands

```
syms s
H(1,1) = s
Z = det(H)
sol = solve(Z)
```

produce

```
H =
[ s, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]
```

```
Z =
s/240 - 1/270
```

```
sol =
8/9
```

Then

```
H = subs(H, s, sol)
```

substitutes the computed value of  $sol$  for  $s$  in  $H$  to give

```
H =
[ 8/9, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]
```

Now, the command

```
det(H)
```

returns

```
ans =
0
```

and

```
inv(H)
```

produces the message

```
ans =  
FAIL
```

because H is singular. For this matrix, null space and column space are nontrivial:

```
Z = null(H)  
C = colspace(H)  
  
Z =  
3/10  
-6/5  
1  
C =  
[ 1, 0]  
[ 0, 1]  
[-3/10, 6/5]
```

It should be pointed out that even though H is singular, `vpa(H)` is not. For any integer value d, setting `digits(d)`, and then computing `inv(vpa(H))` results in an inverse with elements on the order of  $10^d$ .

## Eigenvalues

The symbolic eigenvalues of a square matrix A or the symbolic eigenvalues and eigenvectors of A are computed, respectively, using the commands `E = eig(A)` and `[V,E] = eig(A)`.

The variable-precision counterparts are `E = eig(vpa(A))` and `[V,E] = eig(vpa(A))`.

The eigenvalues of A are the zeros of the characteristic polynomial of A,  $\det(A-x*I)$ , which is computed by `poly(A)`.

The matrix H from the last section provides the first example:

```
H = sym([8/9 1/2 1/3; 1/2 1/3 1/4; 1/3 1/4 1/5])
```

$$\begin{aligned}
 H &= \\
 &[ 8/9, 1/2, 1/3] \\
 &[ 1/2, 1/3, 1/4] \\
 &[ 1/3, 1/4, 1/5]
 \end{aligned}$$

The matrix is singular, so one of its eigenvalues must be zero. The statement

$$[T, E] = \text{eig}(H)$$

produces the matrices T and E. The columns of T are the eigenvectors of H and the diagonal elements of E are the eigenvalues of H:

$$\begin{aligned}
 T &= \\
 &[ 218/285 - (4*12589^{(1/2)})/285, (4*12589^{(1/2)})/285 + 218/285, 3/10] \\
 &[ 292/285 - 12589^{(1/2)}/285, 12589^{(1/2)}/285 + 292/285, -6/5] \\
 &[ 1, 1, 1]
 \end{aligned}$$

$$\begin{aligned}
 E &= \\
 &[ 32/45 - 12589^{(1/2)}/180, 0, 0] \\
 &[ 0, 12589^{(1/2)}/180 + 32/45, 0] \\
 &[ 0, 0, 0]
 \end{aligned}$$

It may be easier to understand the structure of the matrices of eigenvectors, T, and eigenvalues, E, if you convert T and E to decimal notation. To do so, proceed as follows. The commands

$$\begin{aligned}
 Td &= \text{double}(T) \\
 Ed &= \text{double}(E)
 \end{aligned}$$

return

$$\begin{aligned}
 Td &= \\
 &-0.8098 \quad 2.3397 \quad 0.3000 \\
 &0.6309 \quad 1.4182 \quad -1.2000 \\
 &1.0000 \quad 1.0000 \quad 1.0000
 \end{aligned}$$

$$\begin{aligned}
 Ed &= \\
 &0.0878 \quad 0 \quad 0 \\
 &0 \quad 1.3344 \quad 0 \\
 &0 \quad 0 \quad 0
 \end{aligned}$$

The first eigenvalue is zero. The corresponding eigenvector (the first column of  $Td$ ) is the same as the basis for the null space found in the last section. The other two eigenvalues are the result of applying the quadratic formula to

$x^2 - \frac{64}{45}x + \frac{253}{2160}$  which is the quadratic factor in `factor(poly(H))`:

```
syms x
g = simple(factor(poly(H))/x);
solve(g)
```

```
ans =
    32/45 - 12589^(1/2)/180
    12589^(1/2)/180 + 32/45
```

Closed form symbolic expressions for the eigenvalues are possible only when the characteristic polynomial can be expressed as a product of rational polynomials of degree four or less. The Rosser matrix is a classic numerical analysis test matrix that illustrates this requirement. The statement

```
R = sym(rosser)
```

generates

```
R =
[ 611, 196, -192, 407, -8, -52, -49, 29]
[ 196, 899, 113, -192, -71, -43, -8, -44]
[ -192, 113, 899, 196, 61, 49, 8, 52]
[ 407, -192, 196, 611, 8, 44, 59, -23]
[ -8, -71, 61, 8, 411, -599, 208, 208]
[ -52, -43, 49, 44, -599, 411, 208, 208]
[ -49, -8, 8, 59, 208, 208, 99, -911]
[ 29, -44, 52, -23, 208, 208, -911, 99]
```

The commands

```
p = poly(R);
pretty(factor(p))
```

produce



$$x(x - 1020)(x^2 - 1040500)(x^2 - 1020x + 100)(x^2 - 1000)$$

The characteristic polynomial (of degree 8) factors nicely into the product of two linear terms and three quadratic terms. You can see immediately that four of the eigenvalues are 0, 1020, and a double root at 1000. The other four roots are obtained from the remaining quadratics. Use

```
eig(R)
```

to find all these values

```
ans =
      0
     1000
     1000
     1020
510 - 100*26^(1/2)
100*26^(1/2) + 510
(-10)*10405^(1/2)
 10*10405^(1/2)
```

The Rosser matrix is not a typical example; it is rare for a full 8-by-8 matrix to have a characteristic polynomial that factors into such simple form. If you change the two “corner” elements of R from 29 to 30 with the commands

```
S = R; S(1,8) = 30; S(8,1) = 30;
```

and then try

```
p = poly(S)
```

you find

```
p =
x^8 - 4040*x^7 + 5079941*x^6 + 82706090*x^5...
- 5327831918568*x^4 + 4287832912719760*x^3...
- 1082699388411166000*x^2 + 51264008540948000*x...
+ 40250968213600000
```

You also find that `factor(p)` is `p` itself. That is, the characteristic polynomial cannot be factored over the rationals.

For this modified Rosser matrix

```
F = eig(S)
```

returns

```
F =
    1020.420188201504727818545749884
    1019.9935501291629257348091808173
    1019.5243552632016358324933278291
    1000.1206982933841335712817075454
    999.94691786044276755320289228602
    0.21803980548301606860857564424981
   -0.17053529728768998575200874607757
   -1020.05321425589151659318942526
```

Notice that these values are close to the eigenvalues of the original Rosser matrix. Further, the numerical values of `F` are a result of MuPAD software's floating-point arithmetic. Consequently, different settings of `digits` do not alter the number of digits to the right of the decimal place.

It is also possible to try to compute eigenvalues of symbolic matrices, but closed form solutions are rare. The Givens transformation is generated as the matrix exponential of the elementary matrix

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Symbolic Math Toolbox commands

```
syms t
A = sym([0 1; -1 0]);
G = expm(t*A)
```

return

```
G =
[ exp(i*t)/2 + 1/(2*exp(i*t)), i/(2*exp(i*t)) - (i*exp(i*t))/2]
```

$$\left[ \frac{i \exp(i t)}{2} - \frac{i}{2 \exp(i t)}, \quad \frac{\exp(i t)}{2} + \frac{1}{2 \exp(i t)} \right]$$

You can simplify this expression with the `simple` command:

```
[G,how] = simple(G)

G =
[ cos(t), sin(t)]
[ -sin(t), cos(t)]

how =
simplify(100)
```

Next, the command

```
g = eig(G)
```

produces

```
g =
cos(t) - i*sin(t)
cos(t) + i*sin(t)
```

You can use `simple` to simplify this form of `g`:

```
[g,how] = simple(g)

g =
1/exp(i*t)
exp(i*t)

how =
rewrite(exp)
```

## Jordan Canonical Form

The Jordan canonical form results from attempts to diagonalize a matrix by a similarity transformation. For a given matrix  $A$ , find a nonsingular matrix  $V$ , so that  $\text{inv}(V) * A * V$ , or, more succinctly,  $J = V \backslash A * V$ , is “as close to diagonal as possible.” For almost all matrices, the Jordan canonical form is the diagonal matrix of eigenvalues and the columns of the transformation matrix are the eigenvectors. This always happens if the matrix is symmetric

or if it has distinct eigenvalues. Some nonsymmetric matrices with multiple eigenvalues cannot be diagonalized. The Jordan form has the eigenvalues on its diagonal, but some of the superdiagonal elements are one, instead of zero. The statement

```
J = jordan(A)
```

computes the Jordan canonical form of  $A$ . The statement

```
[V,J] = jordan(A)
```

also computes the similarity transformation. The columns of  $V$  are the generalized eigenvectors of  $A$ .

The Jordan form is extremely sensitive to perturbations. Almost any change in  $A$  causes its Jordan form to be diagonal. This makes it very difficult to compute the Jordan form reliably with floating-point arithmetic. It also implies that  $A$  must be known exactly (i.e., without roundoff error, etc.). Its elements must be integers, or ratios of small integers. In particular, the variable-precision calculation, `jordan(vpa(A))`, is not allowed.

For example, let

```
A = sym([12,32,66,116;-25,-76,-164,-294;  
        21,66,143,256;-6,-19,-41,-73])
```

```
A =  
[ 12, 32, 66, 116]  
[ -25, -76, -164, -294]  
[ 21, 66, 143, 256]  
[ -6, -19, -41, -73]
```

Then

```
[V,J] = jordan(A)
```

produces

```
V =  
[ 4, -2, 4, 3]  
[ -6, 8, -11, -8]  
[ 4, -7, 10, 7]
```

$$[ -1, 2, -3, -2]$$

$$\begin{aligned} J = \\ [ 1, 1, 0, 0] \\ [ 0, 1, 0, 0] \\ [ 0, 0, 2, 1] \\ [ 0, 0, 0, 2] \end{aligned}$$

Therefore  $A$  has a double eigenvalue at 1, with a single Jordan block, and a double eigenvalue at 2, also with a single Jordan block. The matrix has only two eigenvectors,  $V(:, 1)$  and  $V(:, 3)$ . They satisfy

$$\begin{aligned} A*V(:, 1) &= 1*V(:, 1) \\ A*V(:, 3) &= 2*V(:, 3) \end{aligned}$$

The other two columns of  $V$  are generalized eigenvectors of grade 2. They satisfy

$$\begin{aligned} A*V(:, 2) &= 1*V(:, 2) + V(:, 1) \\ A*V(:, 4) &= 2*V(:, 4) + V(:, 3) \end{aligned}$$

In mathematical notation, with  $v_j = v(:, j)$ , the columns of  $V$  and eigenvalues satisfy the relationships

$$(A - \lambda_1 I)v_2 = v_1$$

$$(A - \lambda_2 I)v_4 = v_3.$$

## Singular Value Decomposition

Only the variable-precision numeric computation of the complete singular vector decomposition is available in the toolbox. One reason for this is that the formulas that result from symbolic computation are usually too long and complicated to be of much use. If  $A$  is a symbolic matrix of floating-point or variable-precision numbers, then

$$S = \text{svd}(A)$$

computes the singular values of  $A$  to an accuracy determined by the current setting of `digits`. And

```
[U,S,V] = svd(A);
```

produces two orthogonal matrices, U and V, and a diagonal matrix, S, so that

```
A = U*S*V';
```

Consider the n-by-n matrix A with elements defined by  $A(i,j) = 1/(i - j + 1/2)$ . The most obvious way of generating this matrix is

```
n = 5;
for i=1:n
    for j=1:n
        A(i,j) = sym(1/(i-j+1/2));
    end
end
```

For n = 5, the matrix is

```
A
A =
[ 2, -2, -2/3, -2/5, -2/7]
[ 2/3, 2, -2, -2/3, -2/5]
[ 2/5, 2/3, 2, -2, -2/3]
[ 2/7, 2/5, 2/3, 2, -2]
[ 2/9, 2/7, 2/5, 2/3, 2]
```

It turns out many of the singular values of these matrices are close to  $\pi$ .

The most efficient way to generate the matrix is

```
n = 5;
[J,I] = meshgrid(1:n);
A = sym(1./(I - J+1/2));
```

Since the elements of A are the ratios of small integers, vpa(A) produces a variable-precision representation, which is accurate to digits precision. Hence

```
S = svd(vpa(A))
```

computes the desired singular values to full accuracy. With  $n = 16$  and `digits(30)`, the result is

```
S =
 3.14159265358979323846255035973
 3.14159265358979323843066846713
 3.14159265358979323325290142782
 3.14159265358979270342635559052
   3.1415926535897543920684990722
 3.14159265358767361712392612382
 3.14159265349961053143856838564
 3.14159265052654880815569479613
 3.14159256925492306470284863101
 3.14159075458605848728982577118
   3.1415575435991808369105065826
 3.14106044663470063805218371923
 3.13504054399744654843898901261
 3.07790297231119748658424727353
 2.69162158686066606774782763593
 1.20968137605668985332455685355
```

Compare `S` with `pi`, the floating-point representation of  $\pi$ . In the vector below, the first element is computed by subtraction with variable-precision arithmetic and then converted to a double:

```
format long;
double(pi*ones(16,1)-S)
```

The results are

```
ans =
 0.000000000000000
 0.000000000000000
 0.000000000000000
 0.000000000000001
 0.000000000000039
 0.000000000002120
 0.000000000090183
 0.000000003063244
 0.000000084334870
 0.00001899003735
```

```
0.000035109990612
0.000532206955093
0.006552109592347
0.063689681278596
0.449971066729127
1.931911277533103
```

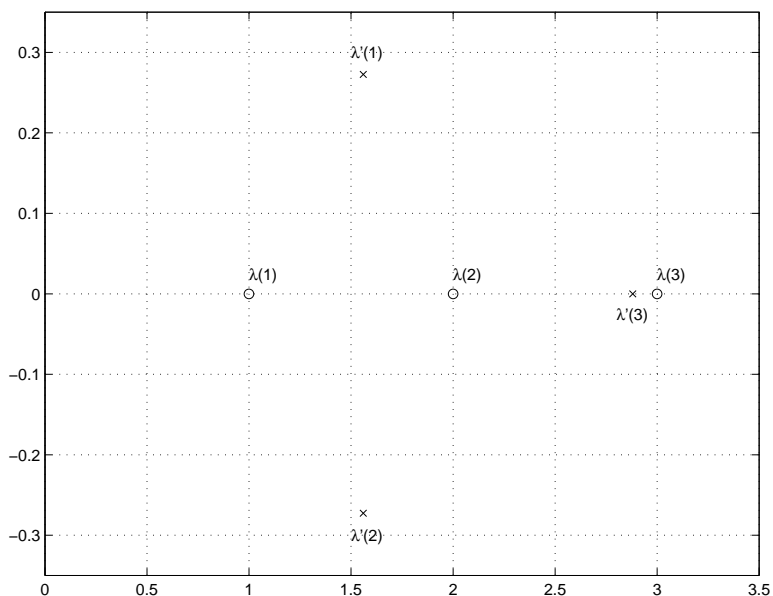
Since the relative accuracy of  $\pi$  is  $\pi * \text{eps}$ , which is  $6.9757\text{e-}16$ , the result confirms the suspicion that four of the singular values of the 16-by-16 example equal  $\pi$  to floating-point accuracy.

## Eigenvalue Trajectories

This example applies several numeric, symbolic, and graphic techniques to study the behavior of matrix eigenvalues as a parameter in the matrix is varied. This particular setting involves numerical analysis and perturbation theory, but the techniques illustrated are more widely applicable.

In this example, you consider a 3-by-3 matrix  $A$  whose eigenvalues are 1, 2, 3. First, you perturb  $A$  by another matrix  $E$  and parameter  $t : A \rightarrow A + tE$ . As  $t$  increases from 0 to  $10^{-6}$ , the eigenvalues  $\lambda_1 = 1$ ,  $\lambda_2 = 2$ ,  $\lambda_3 = 3$  change to  $\lambda_1' = 1.5596 + 0.2726i$ ,  $\lambda_2' = 1.5596 - 0.2726i$ ,  $\lambda_3' = 2.8808$ .





This, in turn, means that for some value of  $t = \tau$ ,  $0 < \tau < 10^{-6}$ , the perturbed matrix  $A(t) = A + tE$  has a double eigenvalue  $\lambda_1 = \lambda_2$ . The example shows how to find the value of  $\tau$ , called  $\tau$ , where this happens.

The starting point is a MATLAB test example, known as `gallery(3)`.

```
A = gallery(3)
```

```
A =
-149    -50   -154
 537    180    546
 -27     -9    -25
```

This is an example of a matrix whose eigenvalues are sensitive to the effects of roundoff errors introduced during their computation. The actual computed eigenvalues may vary from one machine to another, but on a typical workstation, the statements

```
format long
e = eig(A)

produce

e =
    1.000000000010722
    1.99999999991790
    2.99999999997399
```

Of course, the example was created so that its eigenvalues are actually 1, 2, and 3. Note that three or four digits have been lost to roundoff. This can be easily verified with the toolbox. The statements

```
B = sym(A);
e = eig(B) '
p = poly(B)
f = factor(p)

produce

e =
    [1, 2, 3]

p =
    x^3 - 6*x^2 + 11*x - 6

f =
    (x - 3)*(x - 1)*(x - 2)
```

Are the eigenvalues sensitive to the perturbations caused by roundoff error because they are “close together”? Ordinarily, the values 1, 2, and 3 would be regarded as “well separated.” But, in this case, the separation should be viewed on the scale of the original matrix. If  $A$  were replaced by  $A/1000$ , the eigenvalues, which would be .001, .002, .003, would “seem” to be closer together.

But eigenvalue sensitivity is more subtle than just “closeness.” With a carefully chosen perturbation of the matrix, it is possible to make two of its eigenvalues coalesce into an actual double root that is extremely sensitive to roundoff and other errors.

One good perturbation direction can be obtained from the outer product of the left and right eigenvectors associated with the most sensitive eigenvalue. The following statement creates the perturbation matrix:

$$E = [130, -390, 0; 43, -129, 0; 133, -399, 0]$$

$$E = \begin{bmatrix} 130 & -390 & 0 \\ 43 & -129 & 0 \\ 133 & -399 & 0 \end{bmatrix}$$

The perturbation can now be expressed in terms of a single, scalar parameter  $t$ . The statements

```
syms x t
A = A + t*E
```

replace  $A$  with the symbolic representation of its perturbation:

$$A = \begin{bmatrix} 130*t - 149, & -390*t - 50, & -154 \\ 43*t + 537, & 180 - 129*t, & 546 \\ 133*t - 27, & -399*t - 9, & -25 \end{bmatrix}$$

Computing the characteristic polynomial of this new  $A$

```
p = simple(poly(A))
```

gives

$$p = 11*x^3 - 1221271*t*x^2 - x^2*(t + 6) + 492512*t*x + x^3 - 6$$

$p$  is a cubic in  $x$  whose coefficients vary linearly with  $t$ .

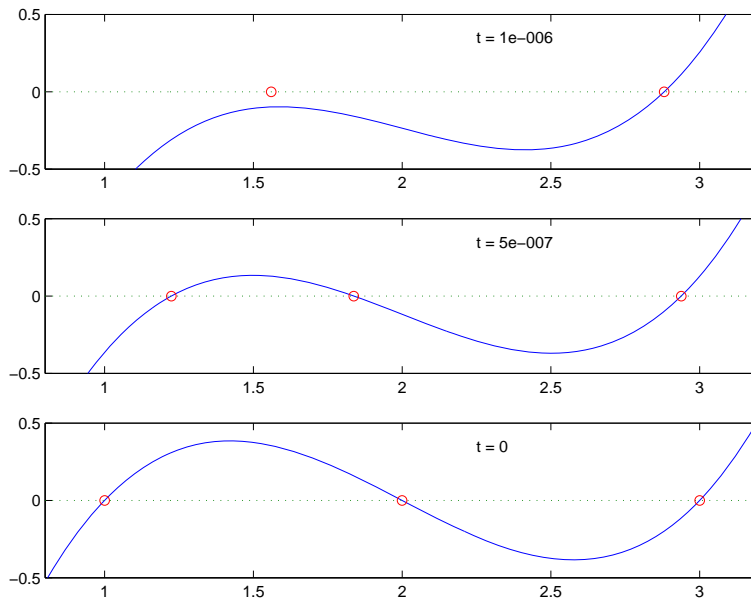
It turns out that when  $t$  is varied over a very small interval, from 0 to  $1.0e-6$ , the desired double root appears. This can best be seen graphically. The first figure shows plots of  $p$ , considered as a function of  $x$ , for three different values of  $t$ :  $t = 0$ ,  $t = 0.5e-6$ , and  $t = 1.0e-6$ . For each value, the eigenvalues are computed numerically and also plotted:

```
x = .8:.01:3.2;
```

```

for k = 0:2
    c = sym2poly(subs(p,t,k*0.5e-6));
    y = polyval(c,x);
    lambda = eig(double(subs(A,t,k*0.5e-6)));
    subplot(3,1,3-k)
    plot(x,y,'-',x,0*x,':',lambda,0*lambda,'o')
    axis([.8 3.2 -.5 .5])
    text(2.25,.35,['t = ' num2str( k*0.5e-6 )]);
end

```



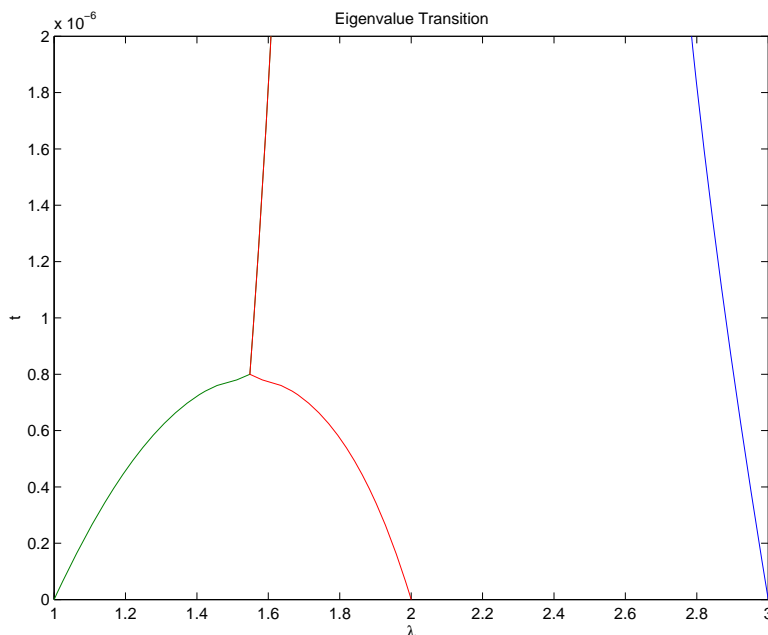
The bottom subplot shows the unperturbed polynomial, with its three roots at 1, 2, and 3. The middle subplot shows the first two roots approaching each other. In the top subplot, these two roots have become complex and only one real root remains.

The next statements compute and display the actual eigenvalues

```
e = eig(A);
```







Above  $\tau = 0.8e^{-6}$ , the graphs of two of the eigenvalues intersect, while below  $\tau = 0.8e^{-6}$ , two real roots become a complex conjugate pair. What is the precise value of  $\tau$  that marks this transition? Let  $\tau$  denote this value of  $\tau$ .

One way to find the *exact* value of  $\tau$  involves polynomial discriminants. The discriminant of a quadratic polynomial is the familiar quantity under the square root sign in the quadratic formula. When it is negative, the two roots are complex.

There is no `discrim` function in the toolbox, but there is one in the MuPAD language. The statement

```
doc(symengine, 'discrim')
```

gives the MuPAD help for the function.

MuPAD documentation

[Provide feedback about this page](#)

## polylib::discrim – discriminant of a polynomial

`polylib::discrim(p, x)` returns the discriminant of the polynomial `p` with respect to the variable `x`.

### → Examples

#### Call:

```
polylib::discrim(p, x)
```

#### Parameters:

This shows that the `discrim` function is in the `polylib` library. Use these commands

```
syms a b c x
evalin(symengine, 'polylib::discrim(a*x^2+b*x+c, x)')
```

to show the generic quadratic's discriminant,  $b^2 - 4ac$ :

```
ans =
b^2 - 4*a*c
```

The discriminant for the perturbed cubic characteristic polynomial is obtained, using

```
discrim = feval(symengine, 'polylib::discrim', p, x)
```

which produces

```
discrim =
242563185060*t^4 - 477857003880091920*t^3 + ...
1403772863224*t^2 - 5910096*t + 4
```

The quantity  $\tau$  is one of the four roots of this quartic. You can find a numeric value for  $\tau$  with the following code.

```
s = solve(discrim);
tau = vpa(s)
```



```
tau =
1970031.04061804553618913725474883634597991201389
0.000000783792490596794010485879469854518820556090553664
0.00000107692481604921513807537160160597784208236311263 - 0.00000308544636502289065492747*i
0.00000308544636502289065492746538275636180217710757295*i + 0.00000107692481604921513807537160160597784249167873707
```

Of the four solutions, you know that

```
tau = tau(2)
```

is the transition point

```
tau =
0.00000078379249059679401048084
```

because it is closest to the previous estimate.

A more generally applicable method for finding  $\tau$  is based on the fact that, at a double root, both the function and its derivative must vanish. This results in two polynomial equations to be solved for two unknowns. The statement

```
sol = solve(p,diff(p,'x'))
```

solves the pair of algebraic equations  $p = 0$  and  $dp/dx = 0$  and produces

```
sol =
t: [4x1 sym]
x: [4x1 sym]
```

Find  $\tau$  now by

```
format short
tau = double(sol.t(2))
```

which reveals that the second element of `sol.t` is the desired value of  $\tau$ :

```
tau =
7.8379e-007
```

Therefore, the second element of `sol.x`

```
sigma = double(sol.x(2))
```

is the double eigenvalue

```
sigma =  
    1.5476
```

To verify that this value of  $\tau$  does indeed produce a double eigenvalue at  $\sigma = 1.5476$ , substitute  $\tau$  for  $t$  in the perturbed matrix  $A(t) = A + tE$  and find the eigenvalues of  $A(t)$ . That is,

```
e = eig(double(subs(A, t, tau)))
```

```
e =  
    1.5476  
    1.5476  
    2.9048
```

confirms that  $\sigma = 1.5476$  is a double eigenvalue of  $A(t)$  for  $t = 7.8379e-07$ .

## Solving Equations

### In this section...

“Solving Algebraic Equations” on page 3-93  
 “Several Algebraic Equations” on page 3-94  
 “Single Differential Equation” on page 3-97  
 “Several Differential Equations” on page 3-100

### Solving Algebraic Equations

If  $S$  is a symbolic expression,

```
solve(S)
```

attempts to find values of the symbolic variable in  $S$  (as determined by `symvar`) for which  $S$  is zero. For example,

```
syms a b c x
S = a*x^2 + b*x + c;
solve(S)
```

uses the familiar quadratic formula to produce

```
ans =
- (b + (b^2 - 4*a*c)^(1/2)) / (2*a)
- (b - (b^2 - 4*a*c)^(1/2)) / (2*a)
```

This is a symbolic vector whose elements are the two solutions.

If you want to solve for a specific variable, you must specify that variable as an additional argument. For example, if you want to solve  $S$  for  $b$ , use the command

```
b = solve(S,b)
```

which returns

```
b =
-(a*x^2 + c)/x
```

Note that these examples assume equations of the form  $f(x) = 0$ . If you need to solve equations of the form  $f(x) = q(x)$ , you must use quoted strings. In particular, the command

```
s = solve('cos(2*x) + sin(x) = 1')
```

returns a vector with three solutions.

```
s =
      0
     pi/6
    (5*pi)/6
```

There are also solutions at each of these results plus  $k\pi$  for integer  $k$ , as you can see in the MuPAD solution:

```
[ solve(cos(2*x) + sin(x) = 1, x)
  { pi*k | k in Z } union { pi/6 + 2*pi*k | k in Z } union { 5*pi/6 + 2*pi*k | k in Z }
```

## Several Algebraic Equations

This section explains how to solve systems of equations using Symbolic Math Toolbox software. As an example, suppose you have the system

$$\begin{aligned} x^2 y^2 &= 0 \\ x - \frac{y}{2} &= \alpha, \end{aligned}$$

and you want to solve for  $x$  and  $y$ . First, create the necessary symbolic objects.

```
syms x y;
alpha = sym('alpha');
```

There are several ways to address the output of `solve`. One is to use a two-output call

```
[x, y] = solve(x^2*y^2, x-y/2 - alpha)
```

which returns

```
x =
  alpha
    0

y =
    0
(-2)*alpha
```

Modify the first equation to  $x^2y^2 = 1$  and there are more solutions.

```
eqs1 = 'x^2*y^2=1, x-y/2-alpha';
[x,y] = solve(eqs1)
```

produces four distinct solutions:

```
x =
alpha/2 + (alpha^2 + 2)^(1/2)/2
alpha/2 + (alpha^2 - 2)^(1/2)/2
alpha/2 - (alpha^2 + 2)^(1/2)/2
alpha/2 - (alpha^2 - 2)^(1/2)/2

y =
(alpha^2 + 2)^(1/2) - alpha
(alpha^2 - 2)^(1/2) - alpha
- alpha - (alpha^2 + 2)^(1/2)
- alpha - (alpha^2 - 2)^(1/2)
```

Since you did not specify the dependent variables, `solve` uses `symvar` to determine the variables.

This way of assigning output from `solve` is quite successful for “small” systems. Plainly, if you had, say, a 10-by-10 system of equations, typing

```
[x1,x2,x3,x4,x5,x6,x7,x8,x9,x10] = solve(...)
```

is both awkward and time consuming. To circumvent this difficulty, `solve` can return a structure whose fields are the solutions. In particular, consider the system  $u^2 - v^2 = a^2$ ,  $u + v = 1$ ,  $a^2 - 2a = 3$ . The command

```
S = solve('u^2 - v^2 = a^2', 'u + v = 1', 'a^2 - 2*a = 3')
```

returns

```
S =  
  a: [2x1 sym]  
  u: [2x1 sym]  
  v: [2x1 sym]
```

The solutions for *a* reside in the “*a*-field” of *S*. That is,

```
S.a
```

produces

```
ans =  
 -1  
  3
```

Similar comments apply to the solutions for *u* and *v*. The structure *S* can now be manipulated by field and index to access a particular portion of the solution. For example, if you want to examine the second solution, you can use the following statement

```
s2 = [S.a(2), S.u(2), S.v(2)]
```

to extract the second component of each field.

```
s2 =  
 [ 3, 5, -4]
```

The following statement

```
M = [S.a, S.u, S.v]
```

creates the solution matrix *M*

```
M =  
 [ -1, 1, 0]  
 [ 3, 5, -4]
```

whose rows comprise the distinct solutions of the system.

Linear systems of simultaneous equations can also be solved using matrix division. For example,

```
clear u v x y
syms u v x y
S = solve(x + 2*y - u, 4*x + 5*y - v);
sol = [S.x; S.y]

A = [1 2; 4 5];
b = [u; v];
z = A\b
```

results in

```
sol =
(2*v)/3 - (5*u)/3
(4*u)/3 - v/3

z =
(2*v)/3 - (5*u)/3
(4*u)/3 - v/3
```

Thus `s` and `z` produce the same solution, although the results are assigned to different variables.

## Single Differential Equation

The function `dsolve` computes symbolic solutions to ordinary differential equations. The equations are specified by symbolic expressions containing the letter `D` to denote differentiation. The symbols `D2`, `D3`, ... `DN`, correspond to the second, third, ..., `N`th derivative, respectively. Thus, `D2y` is the toolbox equivalent of  $d^2y/dt^2$ . The dependent variables are those preceded by `D` and the default independent variable is `t`. Note that names of symbolic variables should not contain `D`. The independent variable can be changed from `t` to some other symbolic variable by including that variable as the last input argument.

Initial conditions can be specified by additional equations. If initial conditions are not specified, the solutions contain constants of integration, `C1`, `C2`, etc.

The output from `dsolve` parallels the output from `solve`. That is, you can call `dsolve` with the number of output variables equal to the number of dependent

variables or place the output in a structure whose fields contain the solutions of the differential equations.

### Example 1

The following call to `dsolve`

```
dsolve('Dy=t*y')
```

uses `y` as the dependent variable and `t` as the default independent variable.

The output of this command is

```
ans =  
C5*exp(t^2/2)
```

$y = C \cdot \exp(t^2/2)$  is a solution to the equation for any constant `C`.

To specify an initial condition, use

```
y = dsolve('Dy=t*y', 'y(0)=2')
```

This produces

```
y =  
2*exp(t^2/2)
```

Notice that `y` is in the MATLAB workspace, but the independent variable `t` is not. Thus, the command `diff(y,t)` returns an error. To place `t` in the workspace, enter `syms t`.

### Example 2

Nonlinear equations may have multiple solutions, even when initial conditions are given:

```
x = dsolve('(Dx+x)^2=1', 'x(0)=0')
```

results in

```
x =  
1/exp(t) - 1
```



$$1 - 1/\exp(t)$$

### Example 3

Here is a second-order differential equation with two initial conditions, and the default independent variable changed to x. The commands

```
y = dsolve('D2y=cos(2*x)-y','y(0)=1','Dy(0)=0','x');
simplify(y, 400)
```

produce

```
ans =
1 - (2*(cos(x) - 1)^2)/3
```

### Example 4

The key issues in this example are the order of the equation and the initial conditions. To solve the ordinary differential equation

$$\frac{d^3u}{dx^3} = u$$

$$u(0) = 1, u'(0) = -1, u''(0) = \pi,$$

with x as the independent variable, type

```
u = dsolve('D3u=u','u(0)=1','Du(0)=-1','D2u(0) = pi','x')
```

Use D3u to represent  $d^3u/dx^3$  and D2u(0) for  $u''(0)$ .

```
u =
(pi*exp(x))/3 - (cos((3^(1/2)*x)/2)*(pi/3 - 1))/exp(x/2) ...
- (3^(1/2)*sin((3^(1/2)*x)/2)*(pi + 1))/(3*exp(x/2))
```

### Further ODE Examples

This table shows a few more examples of differential equations and their Symbolic Math Toolbox syntax. The final entry in the table is the Airy differential equation, whose solution is referred to as the Airy function.

Differential Equation	MATLAB Command
$\frac{dy}{dt} + 4y(t) = e^{-t}$ $y(0) = 1$	<code>y = dsolve('Dy+4*y = exp(-t)', 'y(0) = 1')</code>
$2x^2y'' + 3xy' - y = 0$ $(' = d/dx)$	<code>y = dsolve('2*x^2*D2y + 3*x*Dy - y = 0', 'x')</code>
$\frac{d^2y}{dx^2} = xy(x)$ $y(0) = 0, y(3) = \frac{1}{\pi} K_{1/3}(2\sqrt{3})$ <p>(The Airy equation)</p>	<code>y = dsolve('D2y = x*y', 'y(0) = 0', 'y(3) = bessellk(1/3, 2*sqrt(3))/pi', 'x')</code>

## Several Differential Equations

The function `dsolve` can also handle several ordinary differential equations in several variables, with or without initial conditions. For example, here is a pair of linear, first-order equations.

$$S = \text{dsolve}('Df = 3*f + 4*g', 'Dg = -4*f + 3*g')$$

The computed solutions are returned in the structure `S`. You can determine the values of `f` and `g` by typing

$$f = S.f$$

$$g = S.g$$

$$f =$$

$$C5*\cos(4*t)*\exp(3*t) + C1*\sin(4*t)*\exp(3*t)$$

$$g =$$

$$C1*\cos(4*t)*\exp(3*t) - C5*\sin(4*t)*\exp(3*t)$$

If you prefer to recover `f` and `g` directly as well as include initial conditions, type

```
[f,g] = dsolve('Df=3*f+4*g, Dg =-4*f+3*g',  
'f(0) = 0, g(0) = 1')
```

```
f =  
sin(4*t)*exp(3*t)
```

```
g =  
cos(4*t)*exp(3*t)
```

## Integral Transforms and Z-Transforms

### In this section...

“The Fourier and Inverse Fourier Transforms” on page 3-102

“The Laplace and Inverse Laplace Transforms” on page 3-109

“The Z- and Inverse Z-transforms” on page 3-115

### The Fourier and Inverse Fourier Transforms

The Fourier transform of a function  $f(x)$  is defined as

$$F[f](w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx,$$

and the inverse Fourier transform (IFT) as

$$F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{iwx} du.$$

We refer to this formulation as the Fourier transform of  $f$  with respect to  $x$  as a function of  $w$ . Or, more concisely, the Fourier transform of  $f$  with respect to  $x$  at  $w$ . Mathematicians often use the notation  $F[f]$  to denote the Fourier transform of  $f$ . In this setting, the transform is taken with respect to the independent variable of  $f$  (if  $f = f(t)$ , then  $t$  is the independent variable;  $f = f(x)$  implies that  $x$  is the independent variable, etc.) at the default variable  $w$ . We refer to  $F[f]$  as the Fourier transform of  $f$  at  $w$  and  $F^{-1}[f]$  is the IFT of  $f$  at  $x$ . See `fourier` and `ifourier` in the reference pages for tables that show the Symbolic Math Toolbox commands equivalent to various mathematical representations of the Fourier and inverse Fourier transforms.

For example, consider the Fourier transform of the Cauchy density function,  $(\pi(1 + x^2))^{-1}$ :

```
syms x
cauchy = 1/(pi*(1+x^2));
```

```

fcauchy = fourier(cauchy)

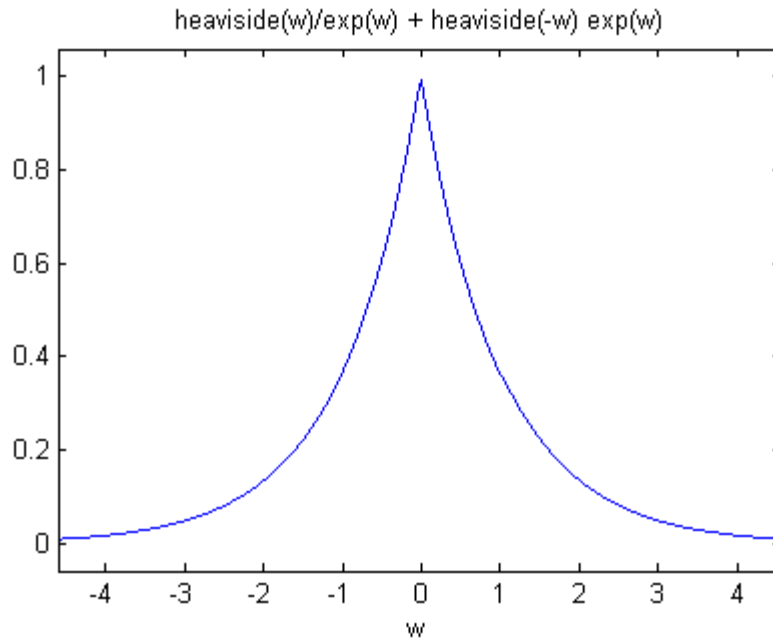
fcauchy =
((pi*heaviside(w))/exp(w) + pi*heaviside(-w)*exp(w))/pi

fcauchy = expand(fcauchy)

fcauchy =
heaviside(w)/exp(w) + heaviside(-w)*exp(w)

ezplot(fcauchy)

```



The Fourier transform is symmetric, since the original Cauchy density function is symmetric.

To recover the Cauchy density function from the Fourier transform, call `ifourier`:

```

finvfcauchy = ifourier(fcauchy)

```

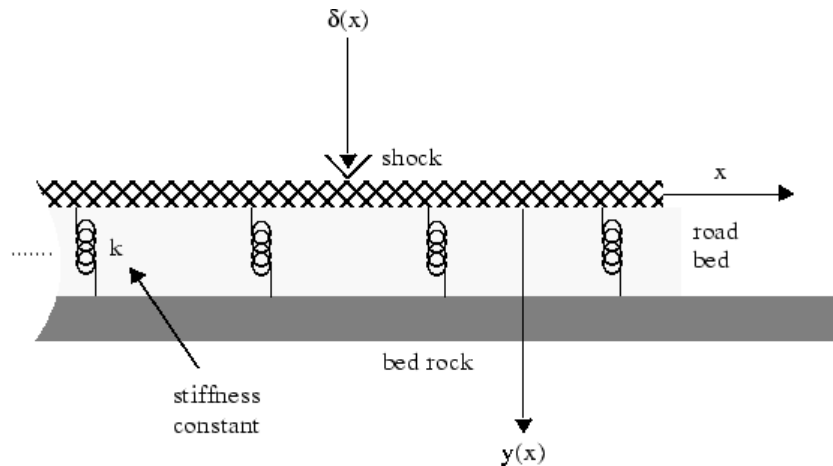
```

finvfcauchy =
-(1/(i*x - 1) - 1/(i*x + 1))/(2*pi)

simplify(finvfcauchy)

ans =
1/(pi*(x^2 + 1))
    
```

An application of the Fourier transform is the solution of ordinary and partial differential equations over the real line. Consider the deformation of an infinitely long beam resting on an elastic foundation with a shock applied to it at a point. A “real world” analogy to this phenomenon is a set of railroad tracks atop a road bed.



The shock could be induced by a pneumatic hammer blow.

The differential equation idealizing this physical setting is

$$\frac{d^4 y}{dx^4} + \frac{k}{EI} y = \frac{1}{EI} \delta(x), \quad -\infty < x < \infty.$$

Here,  $E$  represents elasticity of the beam (rail road track),  $I$  is the “beam constant,” and  $k$  is the spring (road bed) stiffness. The shock force on the right hand side of the differential equation is modeled by the Dirac Delta

function  $\delta(x)$ . If you are unfamiliar with  $\delta(x)$ , you may be surprised to learn that (despite its name), it is not a function at all. Rather,  $\delta(x)$  is an example of what mathematicians call a *distribution*. The Dirac Delta function (named after the physicist Paul Dirac) has the following important property

$$\int_{-\infty}^{\infty} f(x-y)\delta(y)dy = f(x).$$

A definition of the Dirac Delta function is

$$\delta(x) = \lim_{n \rightarrow \infty} n\chi_{(-1/2n, 1/2n)}(x),$$

where

$$\chi_{(-1/2n, 1/2n)}(x) = \begin{cases} 1 & \text{for } -\frac{1}{2n} < x < \frac{1}{2n} \\ 0 & \text{otherwise.} \end{cases}$$

You can evaluate the Dirac Delta function at a point (say)  $x = 3$ , using the commands

```
syms x
del = sym('dirac(x)');
vpa(subs(del, x, 3))
```

which return

```
ans =
0.0
```

Returning to the differential equation, let  $Y(w) = F[y(x)](w)$  and  $\Delta(w) = F[\delta(x)](w)$ . Indeed, try the command `fourier(del, x, w)`. The Fourier transform turns differentiation into exponentiation, and, in particular,

$$F\left[\frac{d^4 y}{dx^4}\right](w) = w^4 Y(w).$$

To see a demonstration of this property, try this

```
syms w x
fourier(diff(sym('y(x)'), x, 4), x, w)
```

which returns

```
ans =
w^4*transform::fourier(y(x), x, -w)
```

Note that you can call the `fourier` command with one, two, or three inputs (see the reference pages for `fourier`). With a single input argument, `fourier(f)` returns a function of the default variable `w`. If the input argument is a function of `w`, `fourier(f)` returns a function of `t`. All inputs to `fourier` must be symbolic objects.

We now see that applying the Fourier transform to the differential equation above yields the algebraic equation

$$\left(w^4 + \frac{k}{EI}\right)Y(w) = \Delta(w),$$

or

$$Y(w) = \Delta(w)G(w),$$

where

$$G(w) = \frac{1}{w^4 + \frac{k}{EI}} = F[g(x)](w)$$

for some function  $g(x)$ . That is,  $g$  is the inverse Fourier transform of  $G$ :

$$g(x) = F^{-1}[G(w)](x)$$

The Symbolic Math Toolbox counterpart to the IFT is `ifourier`. This behavior of `ifourier` parallels `fourier` with one, two, or three input arguments (see the reference pages for `ifourier`).



Continuing with the solution of our differential equation, we observe that the ratio

$$\frac{K}{EI}$$

is a relatively “large” number since the road bed has a high stiffness constant  $k$  and a rail road track has a low elasticity  $E$  and beam constant  $I$ . We make the simplifying assumption that

$$\frac{K}{EI} = 1024.$$

This is done to ease the computation of  $F^{-1}[G(w)](x)$ . Proceeding, we type

```
G = 1/(w^4 + 1024);
g = ifourier(G, w, x);
g = simplify(g);
pretty(g)
```

and see

$$\frac{1}{2} \frac{\sin\left(\frac{\pi}{4} + 4x\right) \operatorname{heaviside}(x)}{\sqrt{4}} + \frac{1}{2} \frac{\operatorname{heaviside}(-x) \sin\left(\frac{\pi}{4} - 4x\right) \exp(4x)}{\sqrt{4}}$$

Notice that  $g$  contains the Heaviside distribution

$$H(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x < 0 \\ 1/2 & \text{for } x = 0. \end{cases}$$

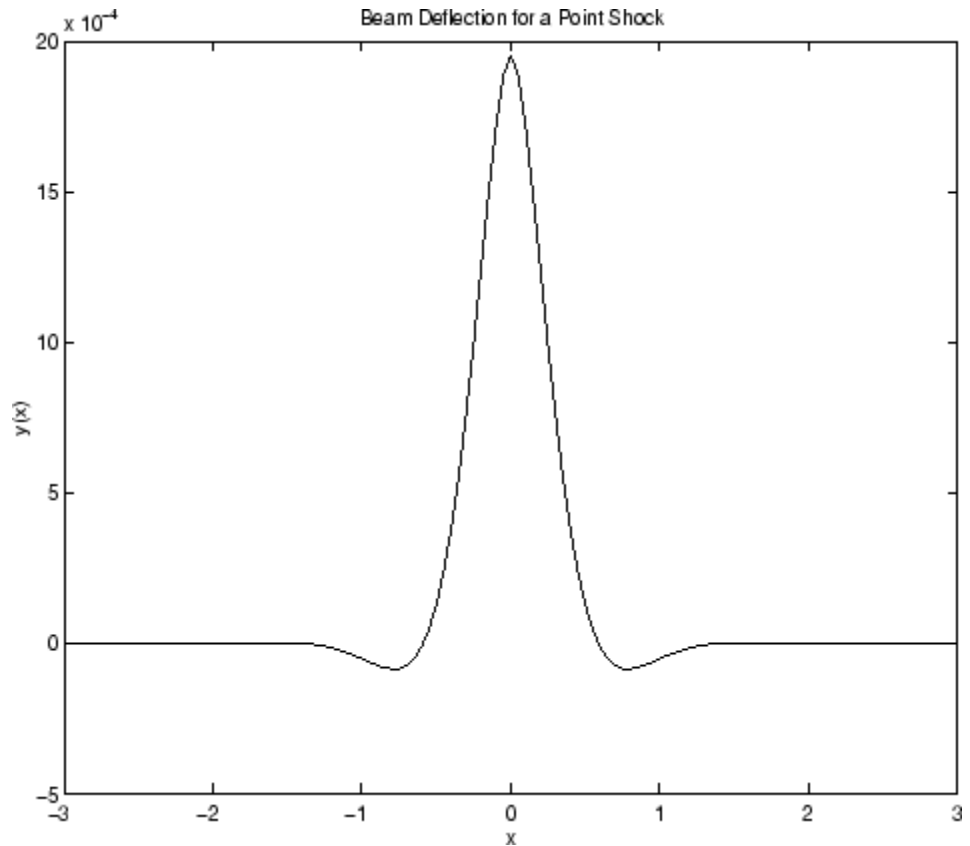
Since  $Y$  is the product of Fourier transforms,  $y$  is the convolution of the transformed functions. That is,  $F[y] = Y(w) = \Delta(w) G(w) = F[\delta] F[g]$  implies

$$y(x) = (\delta * g)(x) = \int_{-\infty}^{\infty} g(x-y)\delta(y)dy = g(x).$$

by the special property of the Dirac Delta function. To plot this function, we must substitute the domain of  $x$  into  $y(x)$ , using the `subs` command.

```
XX = -3:0.05:3;  
YY = double(subs(g, x, XX));  
plot(XX, YY)  
title('Beam Deflection for a Point Shock')  
xlabel('x'); ylabel('y(x)');
```

The resulting graph



shows that the impact of a blow on a beam is highly localized; the greatest deflection occurs at the point of impact and falls off sharply from there. This is the behavior we expect from experience.

## The Laplace and Inverse Laplace Transforms

The Laplace transform of a function  $f(t)$  is defined as

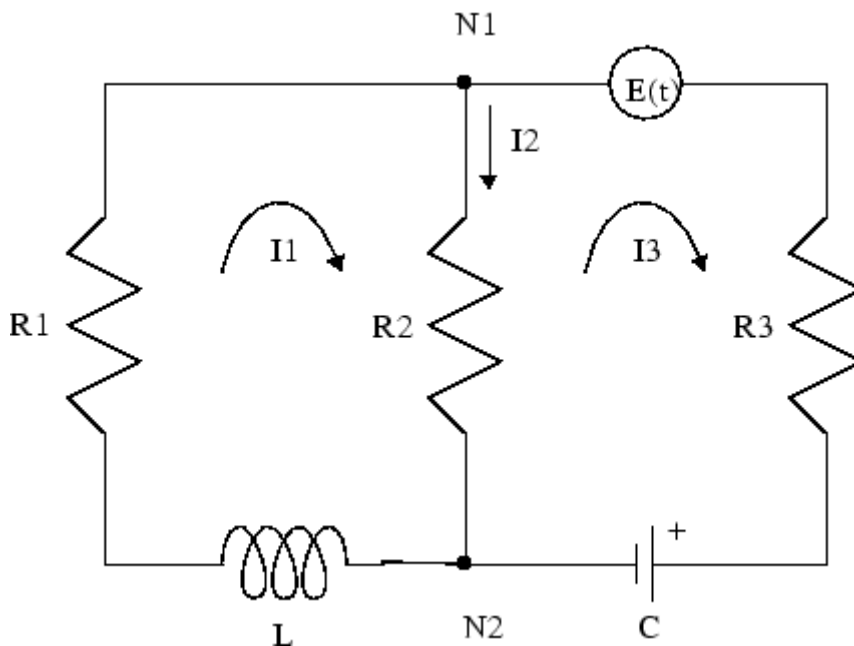
$$L[f](s) = \int_0^{\infty} f(t)e^{-ts} dt,$$

while the inverse Laplace transform (ILT) of  $f(s)$  is

$$L^{-1}[f](t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} f(s)e^{st} ds,$$

where  $c$  is a real number selected so that all singularities of  $f(s)$  are to the left of the line  $s = c$ . The notation  $L[f]$  denotes the Laplace transform of  $f$  at  $s$ . Similarly,  $L^{-1}[f]$  is the ILT of  $f$  at  $t$ .

The Laplace transform has many applications including the solution of ordinary differential equations/initial value problems. Consider the resistance-inductor-capacitor (RLC) circuit below.



Let  $R_j$  and  $I_j$ ,  $j = 1, 2, 3$  be resistances (measured in ohms) and currents (amperes), respectively;  $L$  be inductance (henrys), and  $C$  be capacitance (farads);  $E(t)$  be the electromotive force, and  $Q(t)$  be the charge.

By applying Kirchhoff's voltage and current laws, Ohm's Law, and Faraday's Law, you can arrive at the following system of simultaneous ordinary differential equations.

$$\frac{dI_1}{dt} + \frac{R_2}{L} \frac{dQ}{dt} = \frac{R_1 - R_2}{L} I_1, \quad I_1(0) = I_0.$$

$$\frac{dQ}{dt} = \frac{1}{R_3 + R_2} \left( E(t) - \frac{1}{C} Q(t) \right) + \frac{R_2}{R_3 + R_2} I_1, \quad Q(0) = Q_0.$$

Let's solve this system of differential equations using `laplace`. We will first treat the  $R_j$ ,  $L$ , and  $C$  as (unknown) real constants and then supply values later on in the computation.

```
syms R1 R2 R3 L C real
dI1 = sym('diff(I1(t),t)'); dQ = sym('diff(Q(t),t)');
I1 = sym('I1(t)'); Q = sym('Q(t)');
syms t s
E = sin(t); % Voltage
eq1 = dI1 + R2*dQ/L - (R2 - R1)*I1/L;
eq2 = dQ - (E - Q/C)/(R2 + R3) - R2*I1/(R2 + R3);
```

At this point, we have constructed the equations in the MATLAB workspace. An approach to solving the differential equations is to apply the Laplace transform, which we will apply to `eq1` and `eq2`. Transforming `eq1` and `eq2`

```
L1 = laplace(eq1,t,s)
L2 = laplace(eq2,t,s)
```

returns

```
L1 =
s*laplace(I1(t), t, s) - I1(0)
+ ((R1 - R2)*laplace(I1(t), t, s))/L
- (R2*(Q(0) - s*laplace(Q(t), t, s)))/L

L2 =
s*laplace(Q(t), t, s) - Q(0)
- (R2*laplace(I1(t), t, s))/(R2 + R3) - (C/(s^2 + 1)
- laplace(Q(t), t, s))/(C*(R2 + R3))
```

Now we need to solve the system of equations  $L1 = 0$ ,  $L2 = 0$  for  $\text{laplace}(I1(t), t, s)$  and  $\text{laplace}(Q(t), t, s)$ , the Laplace transforms of  $I_1$  and  $Q$ , respectively. To do this, we need to make a series of substitutions. For the purposes of this example, use the quantities  $R1 = 4 \Omega$  (ohms),  $R2 = 2 \Omega$ ,  $R3 = 3 \Omega$ ,  $C = 1/4$  farads,  $L = 1.6$  H (henrys),  $I1(0) = 15$  A (amperes), and  $Q(0) = 2$  A/sec. Substituting these values in L1

```
syms LI1 LQ
NI1 = subs(L1, {R1,R2,R3,L,C, 'I1(0)', 'Q(0)'}, ...
           {4,2,3,1.6,1/4,15,2})
```

returns

```
NI1 =
s*laplace(I1(t), t, s) + (5*s*laplace(Q(t), t, s))/4
+ (5*laplace(I1(t), t, s))/4 - 35/2
```

The substitution

```
NQ =
subs(L2, {R1,R2,R3,L,C, 'I1(0)', 'Q(0)'}, {4,2,3,1.6,1/4,15,2})
```

returns

```
NQ =
s*laplace(Q(t), t, s) - 1/(5*(s^2 + 1))
+ (4*laplace(Q(t), t, s))/5 - (2*laplace(I1(t), t, s))/5 - 2
```

To solve for  $\text{laplace}(I1(t), t, s)$  and  $\text{laplace}(Q(t), t, s)$ , we make a final pair of substitutions. First, replace the strings ' $\text{laplace}(I1(t), t, s)$ ' and ' $\text{laplace}(Q(t), t, s)$ ' by the syms LI1 and LQ, using

```
NI1 = ...
subs(NI1, {'laplace(I1(t), t, s)', 'laplace(Q(t), t, s)'}, {LI1, LQ})
```

to obtain

```
NI1 =
(5*LI1)/4 + LI1*s + (5*LQ*s)/4 - 35/2
```

Collecting terms

```
NI1 = collect(NI1, LI1)
```

gives

$$\begin{aligned} \text{NI1} = \\ (s + 5/4)*\text{LI1} + (5*\text{LQ}*s)/4 - 35/2 \end{aligned}$$

A similar string substitution

$$\begin{aligned} \text{NQ} = \dots \\ \text{subs}(\text{NQ}, \{ 'laplace(\text{I1}(t), t, s) ', 'laplace(\text{Q}(t), t, s) ' \}, \{ \text{LI1}, \text{LQ} \}) \end{aligned}$$

yields

$$\begin{aligned} \text{NQ} = \\ (4*\text{LQ})/5 - (2*\text{LI1})/5 + \text{LQ}*s - 1/(5*(s^2 + 1)) - 2 \end{aligned}$$

which, after collecting terms,

$$\text{NQ} = \text{collect}(\text{NQ}, \text{LQ})$$

gives

$$\begin{aligned} \text{NQ} = \\ (s + 4/5)*\text{LQ} - (2*\text{LI1})/5 - 1/(5*(s^2 + 1)) - 2 \end{aligned}$$

Now, solving for LI1 and LQ

$$[\text{LI1}, \text{LQ}] = \text{solve}(\text{NI1}, \text{NQ}, \text{LI1}, \text{LQ})$$

we obtain

$$\begin{aligned} \text{LI1} = \\ (300*s^3 + 280*s^2 + 295*s + 280)/(20*s^4 + 51*s^3 + 40*s^2 + 51*s + 20) \end{aligned}$$

$$\begin{aligned} \text{LQ} = \\ (40*s^3 + 190*s^2 + 44*s + 195)/(20*s^4 + 51*s^3 + 40*s^2 + 51*s + 20) \end{aligned}$$

To recover I1 and Q we need to compute the inverse Laplace transform of LI1 and LQ. Inverting LI1

$$\text{I1} = \text{ilaplace}(\text{LI1}, s, t)$$

produces

```
I1 =
(15*(cosh((1001^(1/2)*t)/40)
- (293*1001^(1/2)*sinh((1001^(1/2)*t)/40))/21879)/exp((51*t)/40)
- (5*sin(t))/51
```

Inverting LQ

```
Q = ilaplace(LQ, s, t)
```

yields

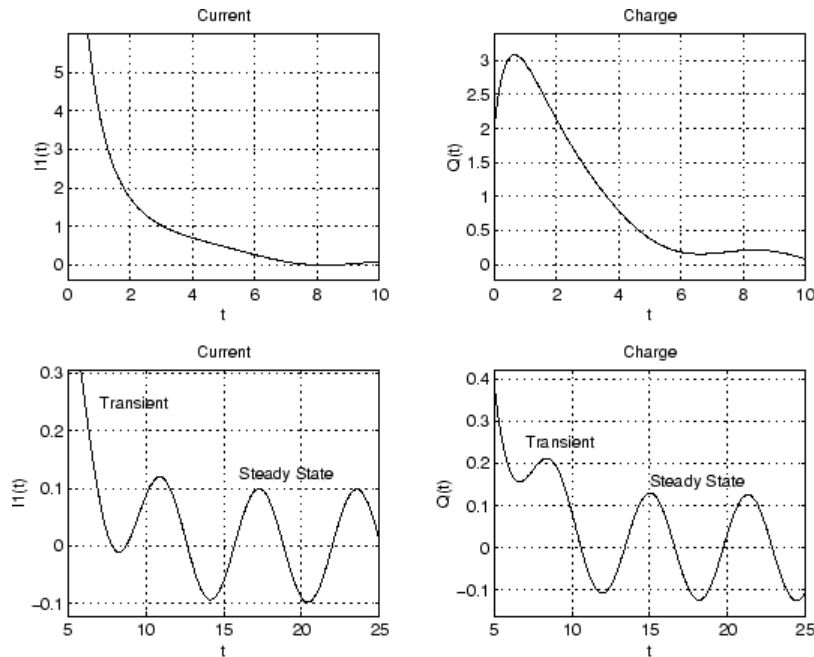
```
Q =
(4*sin(t))/51 - (5*cos(t))/51 + (107*(cosh((1001^(1/2)*t)/40)
+ (2039*1001^(1/2)*sinh((1001^(1/2)*t)/40))/15301)/(51*exp((51*t)/40))
```

Now let's plot the current  $I1(t)$  and charge  $Q(t)$  in two different time domains,  $0 \leq t \leq 10$  and  $5 \leq t \leq 25$ . The statements

```
subplot(2,2,1); ezplot(I1,[0,10]);
title('Current'); ylabel('I1(t)'); grid
subplot(2,2,2); ezplot(Q,[0,10]);
title('Charge'); ylabel('Q(t)'); grid
subplot(2,2,3); ezplot(I1,[5,25]);
title('Current'); ylabel('I1(t)'); grid
text(7,0.25,'Transient'); text(16,0.125,'Steady State');
subplot(2,2,4); ezplot(Q,[5,25]);
title('Charge'); ylabel('Q(t)'); grid
text(7,0.25,'Transient'); text(15,0.16,'Steady State');
```

generate the desired plots





Note that the circuit's behavior, which appears to be exponential decay in the short term, turns out to be oscillatory in the long term. The apparent discrepancy arises because the circuit's behavior actually has two components: an exponential part that decays rapidly (the "transient" component) and an oscillatory part that persists (the "steady-state" component).

## The Z- and Inverse Z-transforms

The (one-sided)  $z$ -transform of a function  $f(n)$  is defined as

$$Z[f](z) = \sum_{n=0}^{\infty} f(n)z^{-n}.$$

The notation  $Z[f]$  refers to the  $z$ -transform of  $f$  at  $z$ . Let  $R$  be a positive number so that the function  $g(z)$  is analytic on and outside the circle  $|z| = R$ . Then the inverse  $z$ -transform (IZT) of  $g$  at  $n$  is defined as

$$Z^{-1}[g](n) = \frac{1}{2\pi i} \oint_{|z|=R} g(z)z^{n-1} dz, \quad n = 1, 2, \dots$$

The notation  $Z^{-1}[f]$  means the IZT of  $f$  at  $n$ . The Symbolic Math Toolbox commands `ztrans` and `iztrans` apply the  $z$ -transform and IZT to symbolic expressions, respectively. See `ztrans` and `iztrans` for tables showing various mathematical representations of the  $z$ -transform and inverse  $z$ -transform and their Symbolic Math Toolbox counterparts.

The  $z$ -transform is often used to solve difference equations. In particular, consider the famous “Rabbit Problem.” That is, suppose that rabbits reproduce only on odd birthdays (1, 3, 5, 7, ...). If  $p(n)$  is the rabbit population at year  $n$ , then  $p$  obeys the difference equation

$$p(n+2) = p(n+1) + p(n), \quad p(0) = 1, \quad p(1) = 2.$$

We can use `ztrans` to find the population each year  $p(n)$ . First, we apply `ztrans` to the equations

```
pn = sym('p(n)');
pn1 = sym('p(n+1)');
pn2 = sym('p(n+2)');
syms n z
eq = pn2 - pn1 - pn;
Zeq = ztrans(eq, n, z)
```

to obtain

```
Zeq =
z*p(0) - z*ztrans(p(n), n, z) - z*p(1) + z^2*ztrans(p(n), n, z)
- z^2*p(0) - ztrans(p(n), n, z)
```

Next, replace `'ztrans(p(n), n, z)'` with `Pz` and insert the initial conditions for  $p(0)$  and  $p(1)$ .

```
syms Pz
Zeq = subs(Zeq, {'ztrans(p(n), n, z)', 'p(0)',
'p(1)'}, {Pz, 1, 2})
```

to obtain

$$\begin{aligned} \text{Zeq} = \\ \text{Pz} * z^2 - z - \text{Pz} * z - \text{Pz} - z^2 \end{aligned}$$

Collecting terms

$$\text{eq} = \text{collect}(\text{Zeq}, \text{Pz})$$

yields

$$\begin{aligned} \text{eq} = \\ (z^2 - z - 1) * \text{Pz} - z^2 - z \end{aligned}$$

Now solve for Pz

$$\text{P} = \text{solve}(\text{eq}, \text{Pz})$$

to obtain

$$\begin{aligned} \text{P} = \\ -(z^2 + z) / (-z^2 + z + 1) \end{aligned}$$

To recover  $p(n)$ , we take the inverse  $z$ -transform of  $P$ .

$$\begin{aligned} \text{p} = \text{iztrans}(\text{P}, z, n); \\ \text{p} = \text{simple}(\text{p}) \end{aligned}$$

The result is a bit complicated, but explicit:

$$\begin{aligned} \text{p} = \\ (3 * 5^{(1/2)} * (1/2 - 5^{(1/2)}/2)^{(n-1)})/5 - \\ (3 * 5^{(1/2)} * (5^{(1/2)}/2 + 1/2)^{(n-1)})/5 + \\ (4 * (-1)^n * \cos(n * (\pi/2 + i * \text{asinh}(1/2))))/i^n \end{aligned}$$

This result can be used as is. But to simplify this result even further, use the MuPAD rewrite function:

$$\begin{aligned} \text{p} = \text{feval}(\text{symengine}, \text{'rewrite'}, \text{p}, \text{'exp'}); \\ \text{p} = \text{simple}(\text{p}) \end{aligned}$$

$$\begin{aligned} \text{p} = \\ (3 * 5^{(1/2)} * (1/2 - 5^{(1/2)}/2)^{(n-1)})/5 - \dots \\ (3 * 5^{(1/2)} * (5^{(1/2)}/2 + 1/2)^{(n-1)})/5 + \dots \\ 2 * \exp(\pi * i * n) * (2 / (5^{(1/2)} + 1))^{n-1} + 2 * (5^{(1/2)}/2 + 1/2)^n \end{aligned}$$

Replace the expression  $\exp(\pi*i*n)$  with  $(-1)^n$ :

```
p = subs(p,exp(pi*i*n),(-1)^n);
p = simple(p)
```

```
p =
((5^(1/2)/2 + 1/2)^n*(3*5^(1/2) + 5) - ...
 (1/2 - 5^(1/2)/2)^n*(3*5^(1/2) + 15) + ...
 20*(-1)^n*(2/(5^(1/2) + 1))^n)/10
```

```
pretty(p)
```

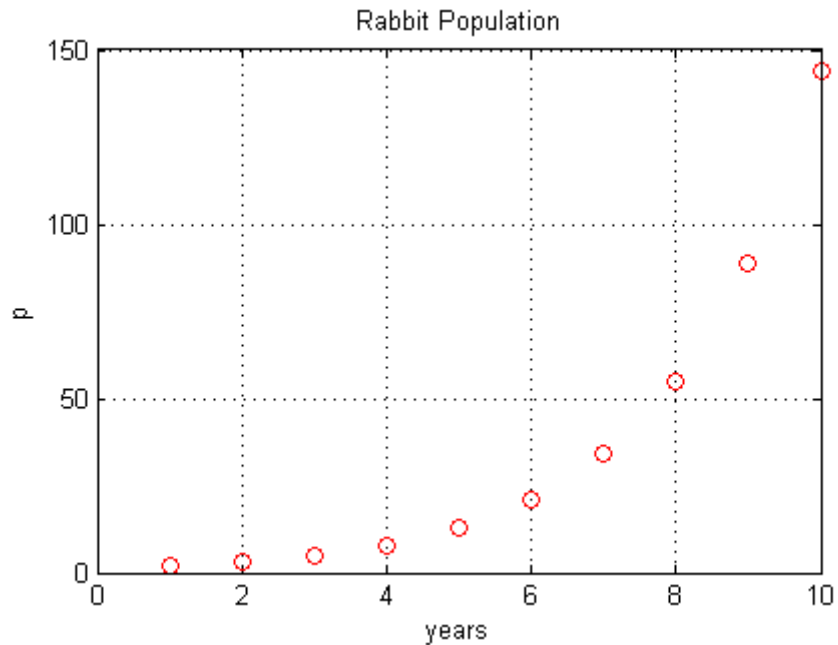
$$\frac{\sqrt{5}^{1/2} \sqrt{n} \left( \sqrt{5}^{1/2} + 1 \right)^n \left( 3 \sqrt{5}^{1/2} + 5 \right) - \left( \sqrt{5}^{1/2} - 1 \right)^n \left( 3 \sqrt{5}^{1/2} + 15 \right) + 20 (-1)^n \left( \frac{2}{\sqrt{5} + 1} \right)^n}{10}$$

10

Finally, plot p:

```
m = 1:10;
y = double(subs(p,n,m));
plot(m,y,'r0')
title('Rabbit Population');
xlabel('years'); ylabel('p');
grid on
```

to show the growth in rabbit population over time.



### References

Andrews, L.C., Shivamoggi, B.K., *Integral Transforms for Engineers and Applied Mathematicians*, Macmillan Publishing Company, New York, 1986

Crandall, R.E., *Projects in Scientific Computation*, Springer-Verlag Publishers, New York, 1994

Strang, G., *Introduction to Applied Mathematics*, Wellesley-Cambridge Press, Wellesley, MA, 1986

## Special Functions of Applied Mathematics

### In this section...

“Numerical Evaluation of Special Functions Using `mfun`” on page 3-120

“Syntax and Definitions of `mfun` Special Functions” on page 3-121

“Diffraction Example” on page 3-126

### Numerical Evaluation of Special Functions Using `mfun`

Over 50 of the special functions of classical applied mathematics are available in the toolbox. These functions are accessed with the `mfun` function, which numerically evaluates special functions for the specified parameters. This allows you to evaluate functions that are not available in standard MATLAB software, such as the Fresnel cosine integral. In addition, you can evaluate several MATLAB special functions in the complex plane, such as the error function `erf`.

For example, suppose you want to evaluate the hyperbolic cosine integral at the points  $2 + i$ ,  $0$ , and  $4.5$ . Look in the tables in “Syntax and Definitions of `mfun` Special Functions” on page 3-121 to find the available functions and their syntax. You can also enter the command

```
mfunlist
```

to see the list of functions available for `mfun`. This list provides a brief mathematical description of each function, its `mfun` name, and the parameters it needs. From the tables or list, you can see that the hyperbolic cosine integral is called `Chi`, and it takes one complex argument.

Type

```
z = [2 + i 0 4.5];  
w = mfun('Chi', z)
```

which returns

```
w =
```

2.0303 + 1.7227i      NaN      13.9658

`mfun` returns NaNs where the function has a singularity. The hyperbolic cosine integral has a singularity at  $z = 0$ .

---

**Note** `mfun` functions perform numerical, not symbolic, calculations. The input parameters should be scalars, vectors, or matrices of type double, or complex doubles, not symbolic variables.

---

## Syntax and Definitions of `mfun` Special Functions

The following conventions are used in the next table, unless otherwise indicated in the **Arguments** column.

$x, y$	real argument
$z, z1, z2$	complex argument
$m, n$	integer argument

### MFUN Special Functions

Function Name	Definition	<code>mfun</code> Name	Arguments
Bernoulli numbers and polynomials	Generating functions: $\frac{e^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \cdot \frac{t^{n-1}}{n!}$	<code>bernoulli(n)</code> <code>bernoulli(n,t)</code>	$n \geq 0$ $0 <  t  < 2\pi$
Bessel functions	<code>BesselI</code> , <code>BesselJ</code> —Bessel functions of the first kind. <code>BesselK</code> , <code>BesselY</code> —Bessel functions of the second kind.	<code>BesselJ(v,x)</code> <code>BesselY(v,x)</code> <code>BesselI(v,x)</code> <code>BesselK(v,x)</code>	$v$ is real.
Beta function	$B(x,y) = \frac{\Gamma(x) \cdot \Gamma(y)}{\Gamma(x+y)}$	<code>Beta(x,y)</code>	

**MFUN Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Binomial coefficients	$\binom{m}{n} = \frac{m!}{n!(m-n)!}$ $= \frac{\Gamma(m+1)}{\Gamma(n+1)\Gamma(m-n+1)}$	binomial(m,n)	
Complete elliptic integrals	Legendre's complete elliptic integrals of the first, second, and third kind	EllipticK(k) EllipticE(k) EllipticPi(a,k)	a is real, $-\infty < a < \infty$ . k is real, $0 < k < 1$ .
Complete elliptic integrals with complementary modulus	Associated complete elliptic integrals of the first, second, and third kind using complementary modulus	EllipticCK(k) EllipticCE(k) EllipticCPi(a,k)	a is real, $-\infty < a < \infty$ . k is real, $0 < k < 1$ .
Complementary error function and its iterated integrals	$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \cdot \int_z^{\infty} e^{-t^2} dt = 1 - \operatorname{erf}(z)$ $\operatorname{erfc}(-1, z) = \frac{2}{\sqrt{\pi}} \cdot e^{-z^2}$ $\operatorname{erfc}(n, z) = \int_z^{\infty} \operatorname{erfc}(n-1, t) dt$	erfc(z) erfc(n, z)	$n > 0$
Dawson's integral	$F(x) = e^{-x^2} \cdot \int_0^x e^{t^2} dt$	dawson(x)	
Digamma function	$\Psi(x) = \frac{d}{dx} \ln(\Gamma(x)) = \frac{\Gamma'(x)}{\Gamma(x)}$	Psi(x)	



**MFUN Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Dilogarithm integral	$f(x) = \int_1^x \frac{\ln(t)}{1-t} dt$	dilog(x)	$x > 1$
Error function	$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$	erf(z)	
Euler numbers and polynomials	Generating function for Euler numbers: $\frac{1}{\cosh(t)} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}$	euler(n) euler(n, z)	$n \geq 0$ $ t  < \frac{\pi}{2}$
Exponential integrals	$Ei(n, z) = \int_1^{\infty} \frac{e^{-zt}}{t^n} dt$ $Ei(x) = PV \left( - \int_{-\infty}^x \frac{e^t}{t} dt \right)$	Ei(n, z) Ei(x)	$n \geq 0$ Real(z) > 0
Fresnel sine and cosine integrals	$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$ $S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$	FresnelC(x) FresnelS(x)	
Gamma function	$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$	GAMMA(z)	
Harmonic function	$h(n) = \sum_{k=1}^n \frac{1}{k} = \Psi(n+1) + \gamma$	harmonic(n)	$n > 0$

**MFUN Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Hyperbolic sine and cosine integrals	$Shi(z) = \int_0^z \frac{\sinh(t)}{t} dt$ $Chi(z) = \gamma + \ln(z) + \int_0^z \frac{\cosh(t) - 1}{t} dt$	Shi(z) Chi(z)	
(Generalized) hypergeometric function	$F(n, d, z) = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^j \Gamma(n_i + k)}{\prod_{i=1}^m \Gamma(d_i)} \cdot \frac{z^k}{k!}$ <p>where j and m are the number of terms in n and d, respectively.</p>	hypergeom(n,d,x) where n = [n1,n2,...] d = [d1,d2,...]	n1,n2,... are real. d1,d2,... are real and nonnegative.
Incomplete elliptic integrals	Legendre's incomplete elliptic integrals of the first, second, and third kind.	EllipticF(x,k) EllipticE(x,k) EllipticPi(x,a,k)	0 < x ≤ ∞. a is real, -∞ < a < ∞. k is real, 0 < k < 1.
Incomplete gamma function	$\Gamma(a, z) = \int_z^{\infty} e^{-t} \cdot t^{a-1} dt$	GAMMA(z1,z2) z1 = a z2 = z	
Logarithm of the gamma function	lnGAMMA(z) = ln(Γ(z))	lnGAMMA(z)	
Logarithmic integral	$Li(x) = PV \left\{ \int_0^x \frac{dt}{\ln t} \right\} = Ei(\ln x)$	Li(x)	x > 1

**MFUN Special Functions (Continued)**

Function Name	Definition	mfun Name	Arguments
Polygamma function	$\Psi^{(n)}(z) = \frac{d^n}{dz} \Psi(z)$ where $\Psi(z)$ is the Digamma function.	Psi(n, z)	$n \geq 0$
Shifted sine integral	$Ssi(z) = Si(z) - \frac{\pi}{2}$	Ssi(z)	

The following orthogonal polynomials are available using mfun. In all cases, n is a nonnegative integer and x is real.

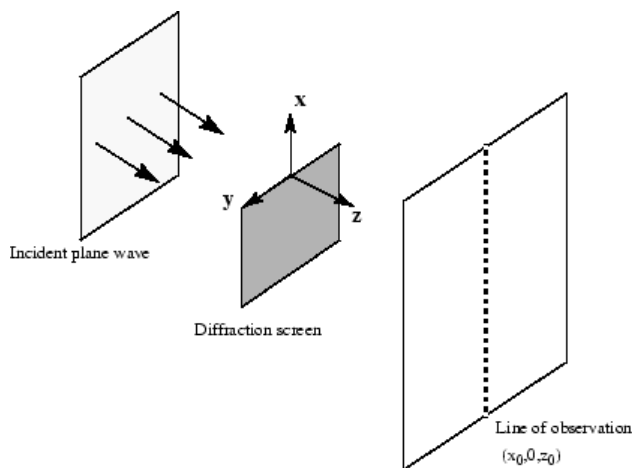
**Orthogonal Polynomials**

Polynomial	mfun Name	Arguments
Chebyshev of the first and second kind	T(n, x) U(n, x)	
Gegenbauer	G(n, a, x)	a is a nonrational algebraic expression or a rational number greater than -1/2.
Hermite	H(n, x)	
Jacobi	P(n, a, b, x)	a, b are nonrational algebraic expressions or rational numbers greater than -1.
Laguerre	L(n, x)	
Generalized Laguerre	L(n, a, x)	a is a nonrational algebraic expression or a rational number greater than -1.
Legendre	P(n, x)	

## Diffraction Example

This example is from diffraction theory in classical electrodynamics. (J.D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, 1962.)

Suppose you have a plane wave of intensity  $I_0$  and wave number  $k$ . Assume that the plane wave is parallel to the  $xy$ -plane and travels along the  $z$ -axis as shown below. This plane wave is called the *incident wave*. A perfectly conducting flat diffraction screen occupies half of the  $xy$ -plane, that is  $x < 0$ . The plane wave strikes the diffraction screen, and you observe the diffracted wave from the line whose coordinates are  $(x, 0, z_0)$ , where  $z_0 > 0$ .



The intensity of the diffracted wave is given by

$$I = \frac{I_0}{2} \left[ \left( C(\zeta) + \frac{1}{2} \right)^2 + \left( S(\zeta) + \frac{1}{2} \right)^2 \right],$$

where

$$\zeta = \sqrt{\frac{k}{2z_0}} \cdot x,$$

and  $C(\zeta)$  and  $S(\zeta)$  are the Fresnel cosine and sine integrals:

$$C(\zeta) = \int_0^\zeta \cos\left(\frac{\pi}{2}t^2\right)dt$$

$$S(\zeta) = \int_0^\zeta \sin\left(\frac{\pi}{2}t^2\right)dt.$$

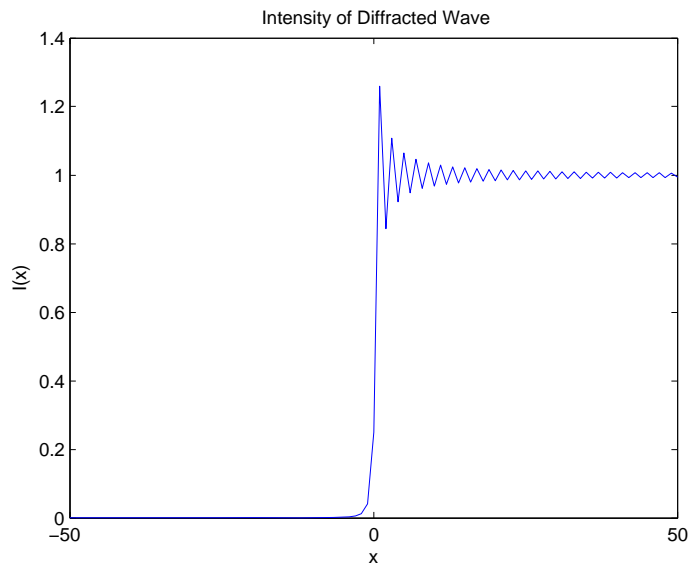
How does the intensity of the diffracted wave behave along the line of observation? Since  $k$  and  $z_0$  are constants independent of  $x$ , you set

$$\sqrt{\frac{k}{2z_0}} = 1,$$

and assume an initial intensity of  $I_0 = 1$  for simplicity.

The following code generates a plot of intensity as a function of  $x$ :

```
x = -50:50;
C = mfun('FresnelC',x);
S = mfun('FresnelS',x);
I0 = 1;
T = (C+1/2).^2 + (S+1/2).^2;
I = (I0/2)*T;
plot(x,I);
xlabel('x');
ylabel('I(x)');
title('Intensity of Diffracted Wave');
```



You see from the graph that the diffraction effect is most prominent near the edge of the diffraction screen ( $x = 0$ ), as you expect.

Note that values of  $x$  that are large and positive correspond to observation points far away from the screen. Here, you would expect the screen to have no effect on the incident wave. That is, the intensity of the diffracted wave should be the same as that of the incident wave. Similarly,  $x$  values that are large and negative correspond to observation points under the screen that are far away from the screen edge. Here, you would expect the diffracted wave to have zero intensity. These results can be verified by setting

$$x = [\text{Inf} \text{ -Inf}]$$

in the code to calculate  $I$ .

## Generating Code from Symbolic Expressions

### In this section...

“Generating C or Fortran Code” on page 3-129

“Generating MATLAB Function Handles and M-Files” on page 3-130

“Generating Embedded MATLAB Function Blocks” on page 3-135

### Generating C or Fortran Code

You can generate C or Fortran code fragments from a symbolic expression, or generate files containing code fragments, using the `ccode` and `fortran` functions. These code fragments calculate numerical values as if substituting numbers for variables in the symbolic expression.

To generate code from a symbolic expression `g`, enter either `ccode(g)` or `fortran(g)`.

For example:

```
syms x y
z = 30*x^4/(x*y^2 + 10) - x^3*(y^2 + 1)^2;
fortran(z)

ans =
    t0 = (x**4*30)/(x*y**2+10)-x**3*(y**2+1)**2

ccode(z)

ans =
    t0 =
    ((x*x*x*x)*3.0E1)/(x*(y*y)+1.0E1)-(x*x*x)*pow(y*y+1.0,2.0);
```

To generate a file containing code, either enter `ccode(g, 'file', 'filename')` or `fortran(g, 'file', 'filename')`. For the example above,

```
fortran(z, 'file', 'fortrantest')
```

generates a file named `fortrantest` in the current directory. `fortrantest` consists of the following:

```
t12 = x**2
t13 = y**2
t14 = t13+1
t0 = (t12**2*30)/(t13*x+10)-t12*t14**2*x
```

Similarly, the command

```
ccode(z,'file','ccodetest')
```

generates a file named `ccodetest` that consists of the lines

```
t16 = x*x;
t17 = y*y;
t18 = t17+1.0;
t0 = ((t16*t16)*3.0E1)/(t17*x+1.0E1)-t16*(t18*t18)*x;
```

`ccode` and `fortran` generate many intermediate variables. This is called *optimized* code. Intermediate variables can make the resulting code more efficient by reusing intermediate expressions (such as `t12` in `fortrantest`, and `t16` in `ccodetest`). They can also make the code easier to read by keeping expressions short.

## Generating MATLAB Function Handles and M-Files

You can use `matlabFunction` to generate a MATLAB function handle that calculates numerical values as if you were substituting numbers for variables in a symbolic expression. You also can use `matlabFunction` to generate an M-file for calculating these numerical values. The generated M-file is available for use in any MATLAB calculation, whether or not the computer running the file has a license for Symbolic Math Toolbox functions.

Symbolic Math Toolbox with a Maple engine does not support `matlabFunction`. For details, see “Differences in Functionality When Using MuPAD and Maple Engines” on page 4-43.



## Generating a Function Handle

`matlabFunction` can generate a function handle from any symbolic expression. For example:

```
syms x y
r = sqrt(x^2 + y^2);
ht = matlabFunction(tanh(r))

ht =
    @(x,y)tanh(sqrt(x.^2+y.^2))
```

You can use this function handle to calculate numerically:

```
ht(.5,.5)

ans =
    0.6089
```

You can pass the usual MATLAB double-precision numbers or matrices to the function handle. For example:

```
cc = [.5,3];
dd = [-.5,.5];
ht(cc, dd)

ans =
    0.6089    0.9954
```

## Controlling the Order of Variables

`matlabFunction` generates input variables in alphabetical order from a symbolic expression. That is why the function handle in “Generating a Function Handle” on page 3-131 has `x` before `y`:

```
ht = @(x,y)tanh((x.^2 + y.^2).^(1./2))
```

You can specify the order of input variables in the function handle using the `vars` option. You specify the order by passing a cell array of strings or symbolic arrays, or a vector of symbolic variables. For example:

```
syms x y z
r = sqrt(x^2 + 3*y^2 + 5*z^2);
```

```
ht1 = matlabFunction(tanh(r), 'vars', [y x z])

ht1 =
    @(y,x,z)tanh(sqrt(x.^2+y.^2.*3.0+z.^2.*5.0))

ht2 = matlabFunction(tanh(r), 'vars', {'x', 'y', 'z'})

ht2 =
    @(x,y,z)tanh(sqrt(x.^2+y.^2.*3.0+z.^2.*5.0))

ht3 = matlabFunction(tanh(r), 'vars', {'x', [y z]})

ht3 =
    @(x,in2)tanh(sqrt(x.^2+in2(:,1).^2.*3.0+in2(:,2).^2.*5.0))
```

### Generating an M-File

You can generate an M-file from a symbolic expression, in addition to a function handle. Specify the file name using the `file` option. Pass a string containing the file name or the path to the file. If you do not specify the path to the file, `matlabFunction` creates this file in the current directory.

This example generates an M-file that calculates the value of the symbolic matrix `F` for double-precision inputs `t`, `x`, and `y`:

```
syms x y t
z = (x^3 - tan(y))/(x^3 + tan(y));
w = z/(1 + t^2);
F = [w, (1 + t^2)*x/y; (1 + t^2)*x/y, 3*z - 1];
matlabFunction(F, 'file', 'testMatrix.m')
```

The file `testMatrix.m` contains the following code:

```
function F = testMatrix(t,x,y)
%TESTMATRIX
%   F = TESTMATRIX(T,X,Y)

%   This function was generated
%   by the Symbolic Math Toolbox version 5.2.
%   07-Nov-2008 12:00:32

t2 = x.^2;
```

```

t3 = tan(y);
t4 = t2.*x;
t5 = t.^2;
t6 = t5 + 1;
t7 = 1./y;
t8 = t6.*t7.*x;
t9 = t3 + t4;
t10 = 1./t9;
F = [-(t10.*(t3 - t4))./t6,t8; t8,- t10.*(3.*t3 - 3.*t2.*x) - 1];

```

`matlabFunction` generates many intermediate variables. This is called *optimized* code. Intermediate variables can make the resulting code more efficient by reusing intermediate expressions (such as `t4`, `t6`, `t8`, `t9`, and `t10` in the calculation of `F`). Using intermediate variables can make the code easier to read by keeping expressions short.

If you don't want the default alphabetical order of input variables, use the `vars` option to control the order. Continuing the example,

```
matlabFunction(F,'file','testMatrix.m','vars',[x y t])
```

generates a file equivalent to the previous one, with a different order of inputs:

```
function F = testMatrix(x,y,t)
...
```

## Naming Output Variables

By default, the names of the output variables coincide with the names you use calling `matlabFunction`. For example, if you call `matlabFunction` with the variable `F`

```

syms x y t
z = (x^3 - tan(y))/(x^3 + tan(y));
w = z/(1 + t^2);
F = [w, (1 + t^2)*x/y; (1 + t^2)*x/y,3*z - 1];
matlabFunction(F,'file','testMatrix.m','vars',[x y t])

```

the generated name of an output variable is also `F`:

```
function F = testMatrix(x,y,t)
...
```

If you call `matlabFunction` using an expression instead of individual variables

```
syms x y t
z = (x^3 - tan(y))/(x^3 + tan(y));
w = z/(1 + t^2);
F = [w, (1 + t^2)*x/y; (1 + t^2)*x/y, 3*z - 1];
matlabFunction(w + z + F, 'file', 'testMatrix.m', ...
'vars', [x y t])
```

the default names of output variables consist of the word `out` followed by the number, for example:

```
function out1 = testMatrix(x,y,t)
...
```

To customize the names of output variables, use the `output` option:

```
syms x y z
r = x^2 + y^2 + z^2;
q = x^2 - y^2 - z^2;
f = matlabFunction(r, q, 'file', 'new_function', ...
'outputs', {'name1', 'name2'})
```

The generated function returns *name1* and *name2* as results:

```
function [name1,name2] = new_function(x,y,z)
...
```

### **Converting MuPAD Expressions**

You can convert a MuPAD expression or function to a MATLAB function:

```
syms x y;
f = evalin(symengine, 'arcsin(x) + arccos(y)');
matlabFunction(f, 'file', 'new_function');
```

The created file contains the same expressions written in the MATLAB language:

```
function f = new_function(x,y)
%NEW_FUNCTION
%   F = NEW_FUNCTION(X,Y)
```

```
% This function was generated by
% the Symbolic Math Toolbox version 5.2.
% 31-Oct-2008 17:41:12

f = asin(x) + acos(y);
```

---

**Note** `matlabFunction` cannot correctly convert some MuPAD expressions to MATLAB functions. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, execute the resulting function.

---

## Generating Embedded MATLAB Function Blocks

Using `emlBlock`, you can generate an Embedded MATLAB™ Function block. The generated block is available for use in Simulink® models, whether or not the computer running the simulations has a license for Symbolic Math Toolbox functions.

Symbolic Math Toolbox with a Maple engine does not support `emlBlock`. For details, see “Differences in Functionality When Using MuPAD and Maple Engines” on page 4-43.

## Generating and Editing a Block

Suppose, you want to create a model involving the van der Pol equation. Before you can convert a symbolic expression to an Embedded MATLAB Function block, create an empty model or open an existing one:

```
new_system('my_system');
open_system('my_system');
```

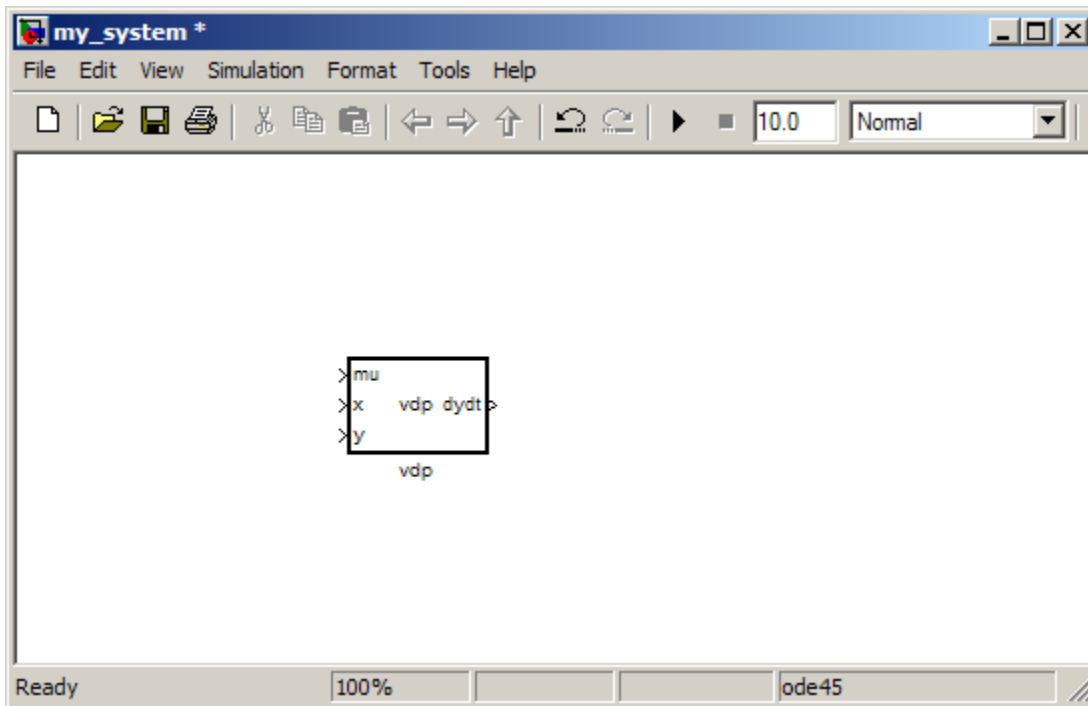
Create a symbolic expression and pass it to the `emlBlock` command. Also specify the block name:

```
syms x y;
mu = sym('mu');
dydt = -x - mu*y*(x^2 - 1);
```

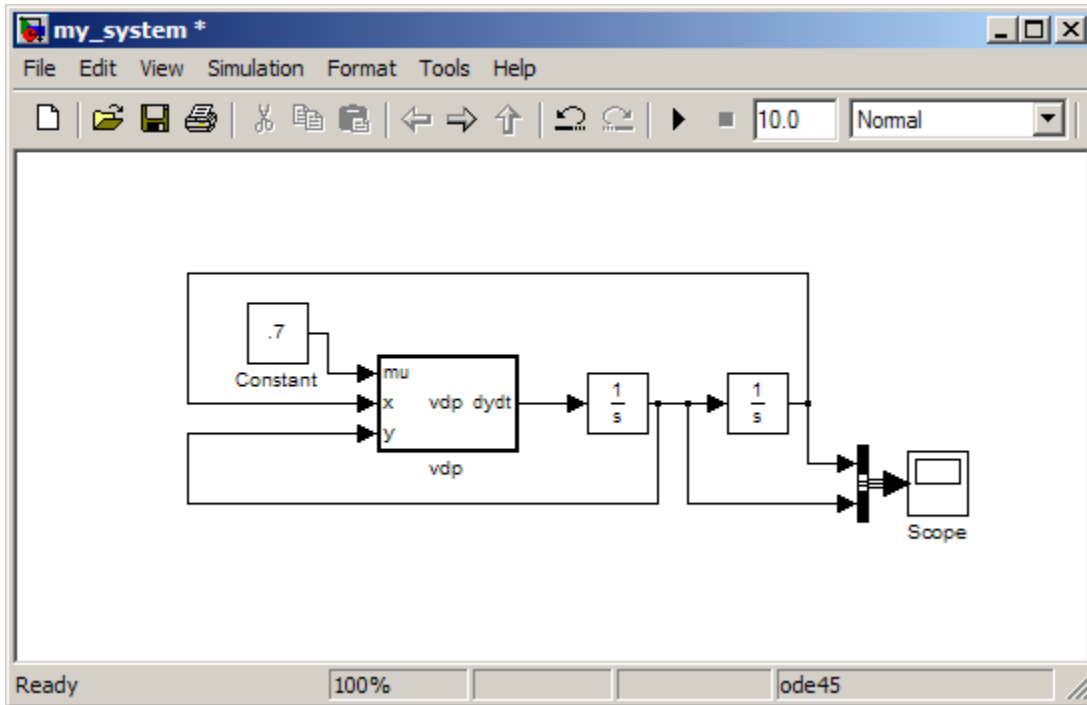
```
emlBlock('my_system/vdp', dydt);
```

If you use the name of an existing block, the `emlBlock` command replaces the definition of an existing block with the converted symbolic expression.

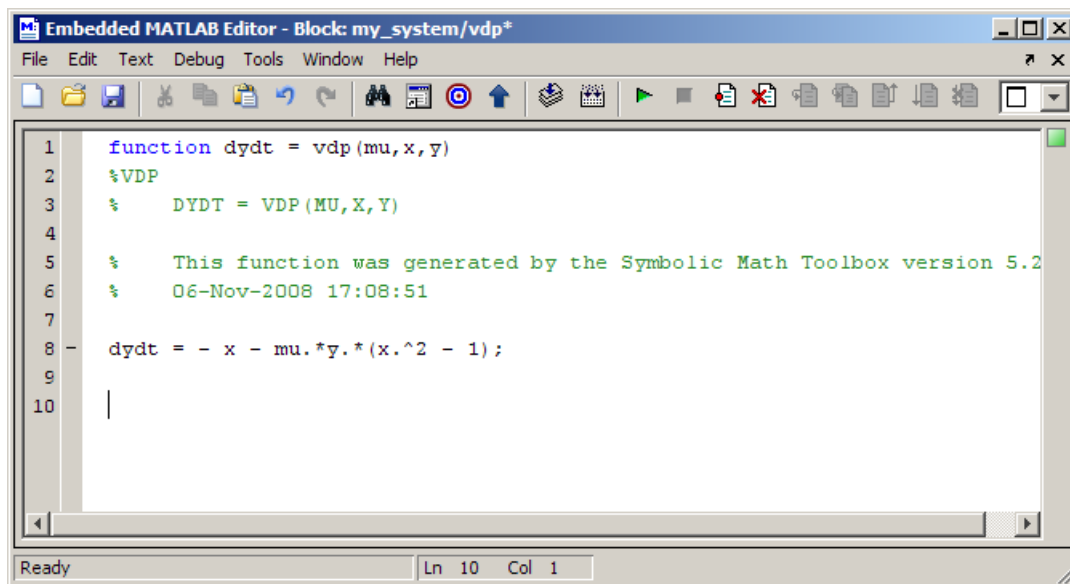
The model `my_system` contains the generated block.



Add other Simulink blocks and wiring to properly define the system.



You can open and edit the generated block. To open a block, select **Edit>Open Block** or use the context menu.



### Controlling the Order of Input Ports

`emlBlock` generates input variables and the corresponding input ports in alphabetical order from a symbolic expression. To change the order of input variables, use the `vars` option:

```

syms x y;
mu = sym('mu');
dydt = -x - mu*y*(x^2 - 1);
emlBlock('my_system/vdp', dydt,...
'vars', [y mu x]);
  
```

### Naming the Output Ports

By default, `emlBlock` generates the names of the output ports as the word `out` followed by the output port number, for example, `out3`. The `output` option allows you to use the custom names of the output ports:

```

syms x y;
  
```



```
mu = sym('mu');
dydt = -x - mu*y*(x^2 - 1);
emlBlock('my_system/vdp', dydt,...
'outputs',{'name1'});
```

## Converting MuPAD Expressions

You can convert a MuPAD expression or function to an Embedded MATLAB Function block:

```
syms x y;
f = evalin(symengine, 'arcsin(x) + arccos(y)');
emlBlock('my_system/my_block', f);
```

The resulting block contains the same expressions written in the MATLAB language:

```
function f = my_block(x,y)
%MY_BLOCK
%   F = MY_BLOCK(X,Y)

%   This function was generated by
%   the Symbolic Math Toolbox version 5.2.
%   31-Oct-2008 11:48:48

f = asin(x) + acos(y);
```

---

**Note** Some MuPAD expressions cannot be correctly converted to a block. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, you can:

- Run the simulation containing the resulting block
  - Open the block and verify that all the functions are defined in the Embedded MATLAB Function Library
-



# MuPAD in Symbolic Math Toolbox

---

- “Understanding MuPAD” on page 4-2
- “MuPAD for MATLAB Users” on page 4-10
- “Integration of MuPAD and MATLAB” on page 4-24
- “Using Maple and MuPAD Engines” on page 4-34

## Understanding MuPAD

### In this section...

“Introduction to MuPAD” on page 4-2

“The MATLAB Workspace and MuPAD Engines” on page 4-2

“Introductory Example Using a MuPAD Notebook from MATLAB” on page 4-3

### Introduction to MuPAD

Version 5 of Symbolic Math Toolbox is powered by the MuPAD symbolic engine.

- Nearly all Symbolic Math Toolbox functions work the same way as in previous versions. To read about the differences with the new engine, see the transition Release Notes.
- MuPAD notebooks provide a new interface for performing symbolic calculations, variable-precision calculations, plotting, and animations. “Introductory Example Using a MuPAD Notebook from MATLAB” on page 4-3 contains an introductory example of how to use this interface.
- Symbolic Math Toolbox functions allow you to copy variables and expressions between the MATLAB workspace and MuPAD notebooks. For more information, see “Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks” on page 4-24.
- You can call MuPAD functions and procedures from the MATLAB environment. For more information, see “Calling MuPAD Functions at the MATLAB Command Line” on page 4-27.

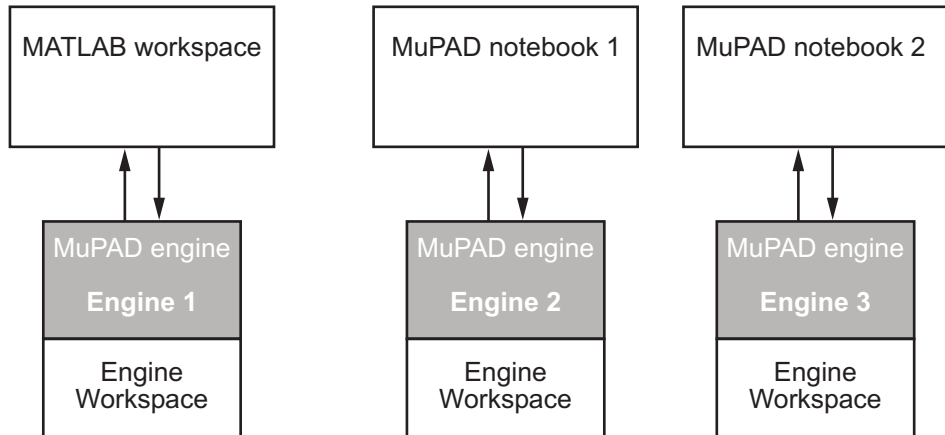
### The MATLAB Workspace and MuPAD Engines

A MuPAD engine is a separate process that runs on your computer in addition to a MATLAB process. A MuPAD engine starts when you first call a function that needs a symbolic engine, such as `syms`. Symbolic Math Toolbox functions that use the symbolic engine use standard MATLAB syntax, such as `y = int(x^2)`.

Conceptually, each MuPAD notebook has its own symbolic engine, with associated workspace. You can have any number of MuPAD notebooks open simultaneously.

One engine exists for use by Symbolic Math Toolbox.

Each MuPAD notebook also has its own engine.



The engine workspace associated with the MATLAB workspace is generally empty, except for assumptions you make about variables. For more information, see “Clearing Assumptions and Resetting the Symbolic Engine” on page 4-29.

## Introductory Example Using a MuPAD Notebook from MATLAB

This example shows how to use a MuPAD notebook to calculate symbolically the mean and variance of a normal random variable that is restricted to be positive. For more information on using a MuPAD notebook, see “Calculating in a MuPAD Notebook” on page 4-14.

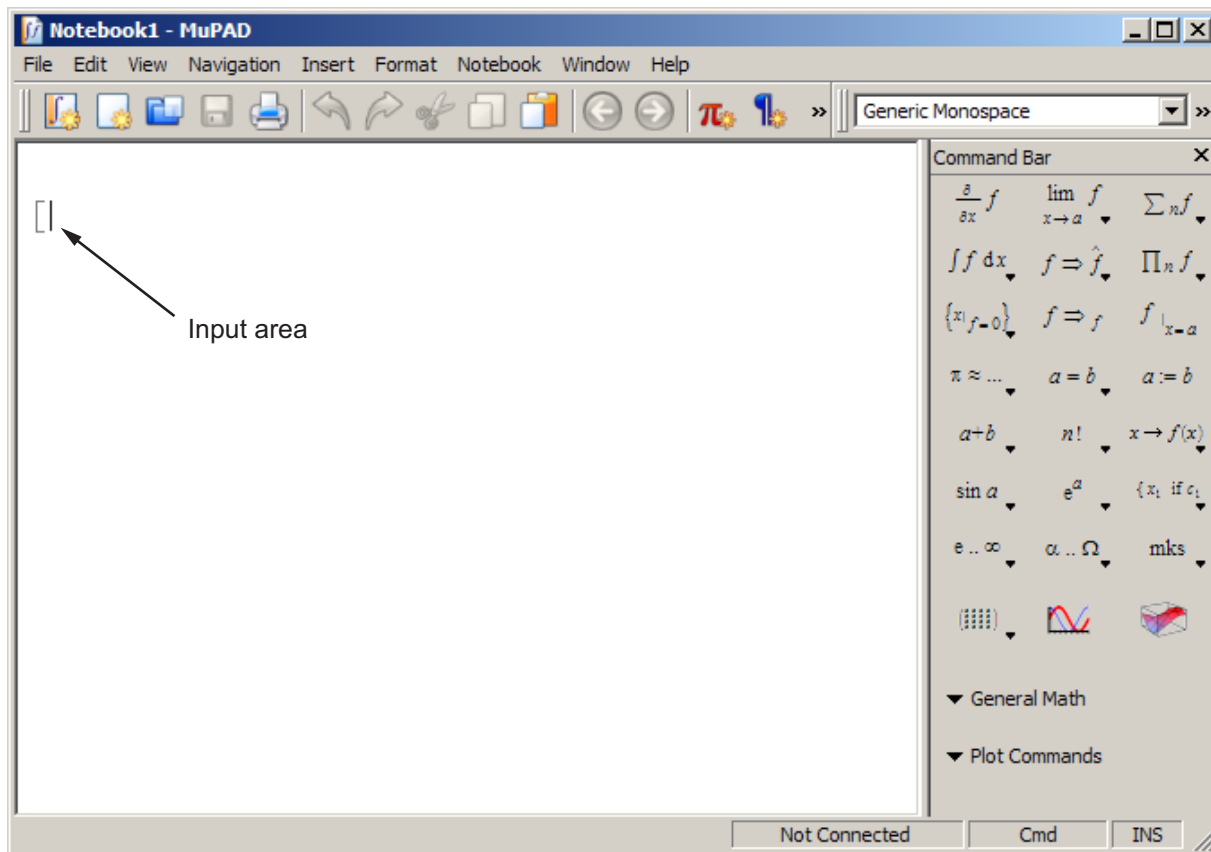
The density function of the normal and positive random variable is

$$f(x) = \begin{cases} e^{-x^2/2} \sqrt{2/\pi} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

1 At the MATLAB command line, enter the command

mupad

2 A blank MuPAD notebook opens. You perform calculations by typing in the input area, demarcated by a left bracket.



3 In the input area, type

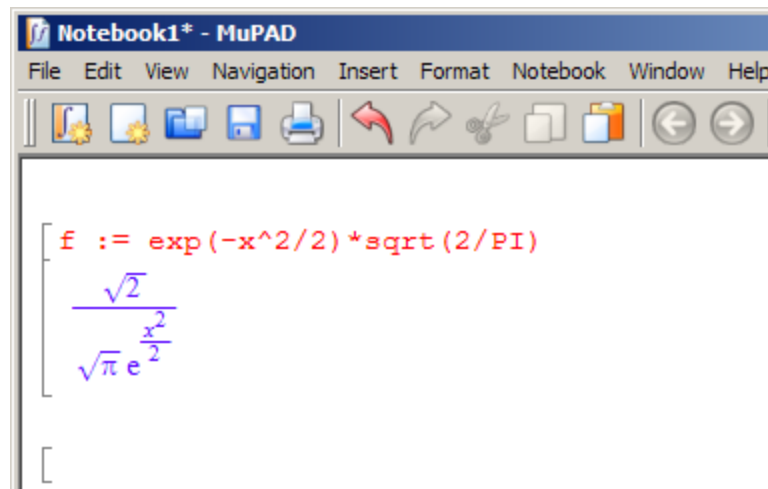
```
f := exp(-x^2/2)*sqrt(2/PI)
```

and press **Enter**.

**Note** Assignment in a MuPAD notebook uses `:=`, not the MATLAB syntax `=`. Also, the MuPAD syntax for the mathematical constant  $\pi$  is `PI`, not the MATLAB syntax `pi`. For more information on common syntax differences, see “Differences Between MATLAB and MuPAD Syntax” on page 4-20.

The MuPAD notebook displays results in real math notation.

Your notebook appears as follows.



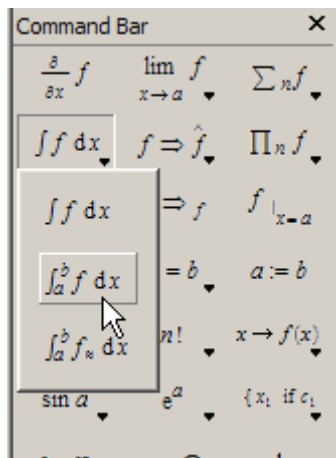
**4** The mean of the random variable is

$$\text{mean} = \int_0^{\infty} x \cdot f dx.$$

To calculate the mean of the random variable, type

`mean :=`

- a** To place an integral in the correct syntax, click the integral button in the right-hand command bar, and select definite limits as shown.



**b** The correct syntax for integration appears in the input area.

```
[mean := int(#f, #x=#a..#b)
```

**c** Replace #f with x\*f, #x with x, #a with 0, and #b with infinity.

- Use the **Tab** key to select the replaceable fields #f, #x, etc.
- Use **Ctrl+space** to autocomplete inputs; e.g., enter `infi` followed by **Ctrl+space** to enter `infinity`.

Once your input area reads

```
mean := int(x*f, x=0..infinity)
```

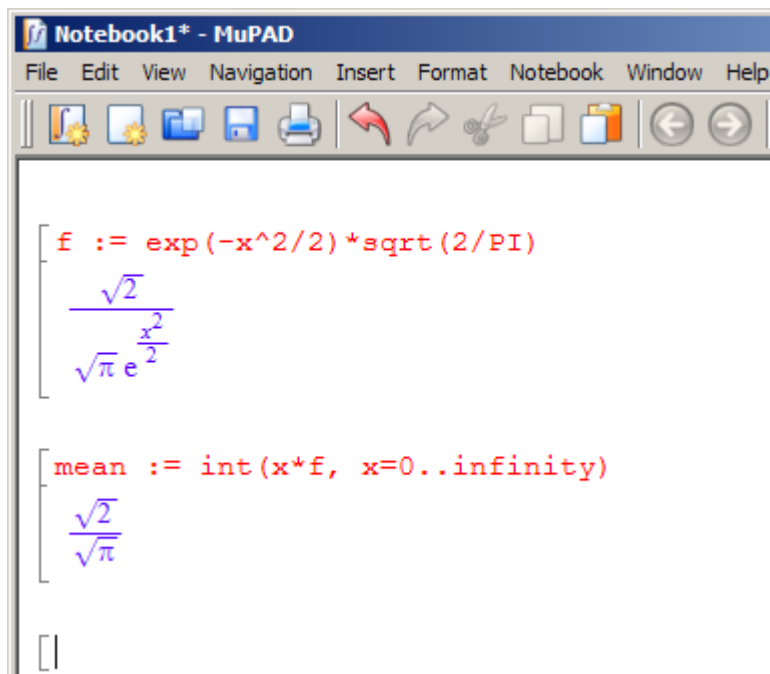
press **Enter**.

---

**Note** The syntax for integration, and for infinity, differ from the MATLAB versions.

---





**5** The variance of the random variable is

$$\text{variance} = \int_0^{\infty} (x - \text{mean})^2 \cdot f dx.$$

To calculate the variance of the random variable, type

```
variance := int((x-mean)^2*f, x=0..infinity)
```

and press **Enter**.

$$\left[ \begin{array}{l} \text{variance} := \text{int}((x-\text{mean})^2*f, x=0..\text{infinity}) \\ \int_0^{\infty} \frac{\sqrt{2} \left(x - \frac{\sqrt{2}}{\sqrt{\pi}}\right)^2}{\sqrt{\pi} e^{\frac{x^2}{2}}} dx \end{array} \right.$$

- 6 The result of evaluating `variance` is a complicated expression. Try to simplify it with the `simplify` command. Type

```
simplify(variance)
```

and press **Enter**. The result is indeed simpler.

$$\left[ \begin{array}{l} \text{simplify}(\text{variance}) \\ \frac{\pi - 2}{\pi} \end{array} \right.$$

- 7 Another expression for the variance of the random variable is

$$\text{variance} = \int_0^{\infty} x^2 \cdot f dx - \text{mean}^2.$$

To calculate the variance of the random variable using this definition, type

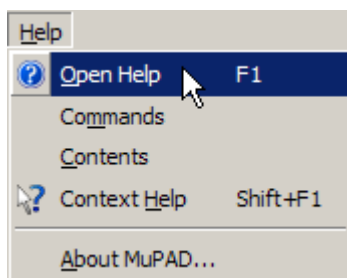
```
variance2 := int(x^2*f, x=0..infinity) - mean^2
```

and press **Enter**.

$$\left[ \begin{array}{l} \text{variance2} := \text{int}(x^2*f, x=0..\text{infinity}) - \text{mean}^2 \\ 1 - \frac{2}{\pi} \end{array} \right.$$

The two expressions for variance, `variance` and `variance2`, are obviously equivalent.

For more information on working in MuPAD notebooks, select **Help > Open Help**, or press **F1** to launch the MuPAD Help viewer.



Within the MuPAD Help viewer, both the “Getting Started” and “The MuPAD Notebook Interface” sections can help you understand and use MuPAD notebooks.

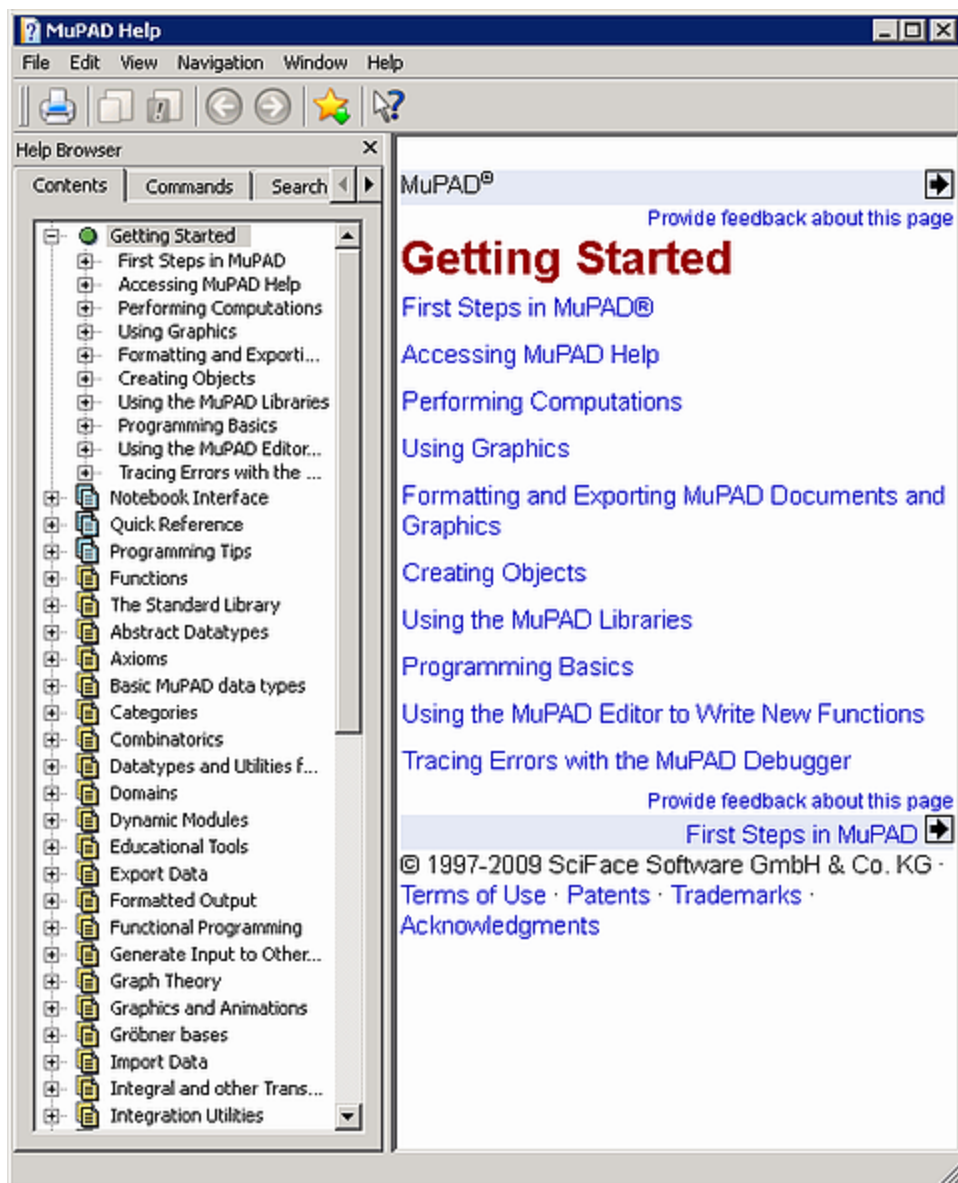
## MuPAD for MATLAB Users

In this section...
“Getting Help for MuPAD” on page 4-10
“Launching, Opening, and Saving MuPAD Notebooks” on page 4-12
“Opening Recent Files and Other MuPAD Interfaces” on page 4-13
“Calculating in a MuPAD Notebook” on page 4-14
“Differences Between MATLAB and MuPAD Syntax” on page 4-20

### Getting Help for MuPAD

There is extensive online help available for MuPAD. You can access the help browser from the MATLAB workspace in a variety of ways:

- Enter `doc(symengine)` at the MATLAB command line to display MuPAD Help.



MuPAD Help contains complete documentation of the MuPAD language. It also explains how to use the various MuPAD interfaces, such as notebooks and the editor.

- For help on a specific MuPAD function, enter `doc(symengine, 'functionName')` at the MATLAB command line to display MuPAD Help at the `functionName` function.
- There is also a MuPAD Tutorial PDF file available at [http://www.mathworks.com/access/helpdesk/...help/pdf\\_doc/symbolic/mupad\\_tutorial.pdf](http://www.mathworks.com/access/helpdesk/...help/pdf_doc/symbolic/mupad_tutorial.pdf).

## Launching, Opening, and Saving MuPAD Notebooks

To open a new MuPAD notebook from the MATLAB command line, enter

```
nb = mupad
```

(You can use any variable name you like instead of `nb`.) This opens a blank MuPAD notebook.

The variable `nb` is a handle to the notebook. This handle is used only for communication between the MATLAB workspace and the MuPAD notebook. It can be used as described in “Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks” on page 4-24.

You can also open an existing MuPAD notebook file named *file\_name* from the MATLAB command line by entering the command

```
nb2 = mupad('file_name')
```

This command is useful in case you lose the handle to a notebook; save the notebook file, and reopen it with a fresh handle.

**Warning** You can lose data when saving a MuPAD notebook. A notebook saves its inputs and outputs, but not the state of its engine. In particular, variables copied into a notebook using `setVar(nb, ...)` are not saved with the notebook.

You can open and save MuPAD notebook files using the usual file system commands, and by using the MATLAB or MuPAD **File** menu. However, to

use a handle to a notebook, you must open the notebook using the `mupad` command at the MATLAB command line.

---

**Note** MuPAD notebook files open in an unevaluated state; in other words, the notebook is not synchronized with its engine when it opens. To synchronize a notebook with its engine, choose **Evaluate All** from the **Notebook** menu. For more information, see “Synchronizing a Notebook and its Engine” on page 4-18.

---

## Opening Recent Files and Other MuPAD Interfaces

If you have no MuPAD interfaces open, the command

```
mupadwelcome
```

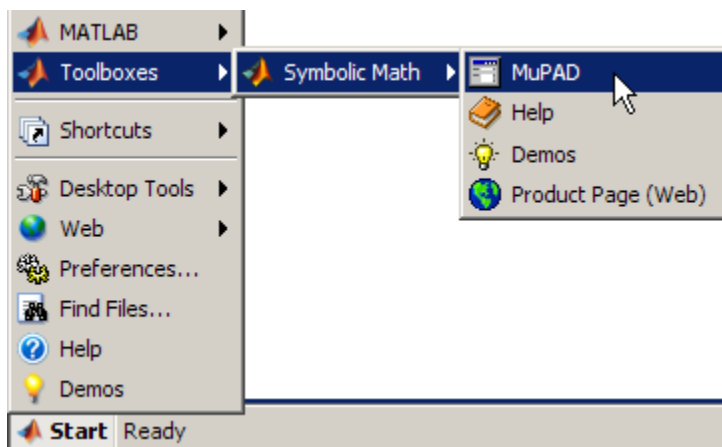
brings up a window for launching various MuPAD interfaces.



- To access MuPAD Help, click one of the three options in the **First Steps** pane.

- To launch a file in the **Open recent File** list, single-click its name.
- To launch a new notebook, click the **New Notebook** button.
- To launch a program editor, click the **New Editor** button. For information on this interface and its associated debugger, see MuPAD Help.
- To open an existing MuPAD notebook or program file, click **Open File** and navigate to the file.

Alternatively, you can launch the mupadwelcome screen from the MATLAB **Start** menu as pictured.



## Calculating in a MuPAD Notebook

### The Visual Elements of a Notebook

A MuPAD notebook has the following main components.



The screenshot shows the MuPAD Notebook1+ interface. At the top is a menu bar with options: File, Edit, View, Navigation, Insert, Format, Notebook, Window, Help. Below the menu is a toolbar with various icons. The main workspace is divided into several sections:

- Input areas:** The top section contains two input areas. The first contains the command `variance := int(x^2*f, x=0..infinity) - mean^2`. The second contains the command `plot(piecewise([x > 0, f], [x <= 0, 0]))`. A blue mathematical expression  $1 - \frac{2}{\pi}$  is shown above the second input area.
- Results and Plots:** The middle section displays a plot of a probability density function. The x-axis ranges from -5 to 5, and the y-axis ranges from 0 to 0.7. The plot shows a curve that is zero for  $x < 0$  and decays from approximately 0.7 at  $x = 0$  towards zero as  $x$  increases.
- Text area:** Below the plot, there is a text area containing the sentence "This is a plot of the density".
- Command Bar:** On the right side, there is a Command Bar with a list of mathematical symbols and functions, such as  $\frac{d}{dx} f$ ,  $\lim_{x \rightarrow a} f$ ,  $\sum_n f$ ,  $\int f dx$ ,  $f \Rightarrow \hat{f}$ ,  $\prod_n f$ ,  $\{x_i, f=0\}$ ,  $f \Rightarrow f$ ,  $f|_{x=a}$ ,  $\pi \approx \dots$ ,  $a = b$ ,  $a := b$ ,  $a+b$ ,  $n!$ ,  $x \rightarrow f(x)$ ,  $\sin a$ ,  $e^a$ ,  $\{x_i \text{ if } c_i$ ,  $e \dots \infty$ ,  $\alpha \dots \Omega$ , and  $mks$ . It also has sections for "General Math" and "Plot Commands".

Arrows from the labels "Menu", "Input areas", "Results and Plots", "Text area", and "Command Bar" point to their respective components in the screenshot.

- Enter commands for execution, evaluation, or plotting in input areas.
- Enter commentary in text areas.
- Use the Command Bar to help you enter commands into input areas with the proper syntax.
- Use the **Insert** menu to add a text area (called **Text Paragraph**) or input area (called **Calculation**).
- Use the **Notebook** menu to evaluate expressions in input areas.

## Working in a Notebook

The MuPAD notebook interface differs from the MATLAB interface. Here are some things to keep in mind when working in a MuPAD notebook:

- Commands typed in an input area are not evaluated until you press **Enter**.
- You can edit the commands typed in *any* input area. For example, you can change a command, correct syntax, or try different values of parameters simply by selecting the area you wish to change and typing over it. Press **Enter** to have the notebook evaluate the result.
- Results do not automatically cascade or propagate through a notebook, as described in “Cascading Calculations” on page 4-16.
- The MATLAB method of recalling a previous command by typing an **up arrow** key does not have the same effect in a MuPAD notebook. Instead, you use arrow keys for navigation in MuPAD notebooks, similar to most word processors.

## Cascading Calculations

If you change a variable in a notebook, the changes do not automatically propagate throughout the notebook. For example, consider the following set of MuPAD commands.

```

z := sin(x)
sin(x)

y := z/(1+z^2)
sin(x)
-----
sin(x)^2 + 1

w := simplify(y/(1-y))
2 sin(x)
-----
cos(2 x) + 2 sin(x) - 3

```

Now change the definition of  $z$  in the first line of the notebook from  $\sin(x)$  to  $\cos(x)$  and press **Enter**.

```

z := cos(x)
cos(x)

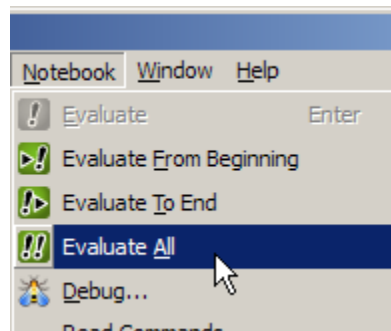
y := z/(1+z^2)
sin(x)
-----
sin(x)^2 + 1

w := simplify(y/(1-y))
- 2 sin(x)
-----
cos(2 x) + 2 sin(x) - 3

```

Only the first line was reevaluated. Therefore  $y$  and  $z$  are no longer synchronized; the notebook is in an inconsistent state.

To have the changes cascade to all parts of the notebook, select **Notebook > Evaluate All**.



The engine evaluates all the expressions in the notebook from top to bottom, and the notebook becomes consistent.

$$\left[ \begin{array}{l} z := \cos(x) \\ \cos(x) \\ \\ y := z / (1+z^2) \\ \frac{\cos(x)}{\cos(x)^2 + 1} \\ \\ w := \text{simplify}(y / (1-y)) \\ \frac{\cos(x)}{\cos(x)^2 - \cos(x) + 1} \end{array} \right.$$

### Synchronizing a Notebook and its Engine

When you open a saved MuPAD notebook file, the notebook display is not synchronized with its engine. For example, suppose you saved the notebook pictured in the start of “Cascading Calculations” on page 4-16:

$$\left[ \begin{array}{l} z := \sin(x) \\ \sin(x) \\ \\ y := z / (1+z^2) \\ \frac{\sin(x)}{\sin(x)^2 + 1} \\ \\ w := \text{simplify}(y / (1-y)) \\ - \frac{2 \sin(x)}{\cos(2x) + 2 \sin(x) - 3} \end{array} \right.$$

If you open that file and immediately try to work in it, without synchronizing the notebook with its engine, the expressions in the notebook display are unavailable for calculations. For example, try to calculate  $u := (1+w)/w$ :

$$\left[ \begin{array}{l} z := \sin(x) \\ \sin(x) \end{array} \right.$$

$$\left[ \begin{array}{l} y := z/(1+z^2) \\ \frac{\sin(x)}{\sin(x)^2 + 1} \end{array} \right.$$

$$\left[ \begin{array}{l} w := \text{simplify}(y/(1-y)) \\ -\frac{2 \sin(x)}{\cos(2x) + 2 \sin(x) - 3} \end{array} \right.$$

$$\left[ \begin{array}{l} u := (1+w)/w \\ \frac{w+1}{w} \end{array} \right.$$

The variable  $w$  has no definition as far as the engine is concerned.

To remedy this situation, select **Notebook > Evaluate All**. The variable  $u$  changes to reflect the value of  $w$ .

$$\left[ \begin{array}{l} u := (1+w)/w \\ \frac{\left( \frac{2 \sin(x)}{\cos(2x) + 2 \sin(x) - 3} - 1 \right) (\cos(2x) + 2 \sin(x) - 3)}{2 \sin(x)} \end{array} \right.$$

## Differences Between MATLAB and MuPAD Syntax

There are several differences between MATLAB and MuPAD syntax. Be aware of which interface you are using in order to use the correct syntax:

- Use MATLAB syntax in the MATLAB workspace, *except* for the functions `evalin(symengine, ...)` and `feval(symengine, ...)`, which use MuPAD syntax.
- Use MuPAD syntax in MuPAD notebooks.

You must define MATLAB variables before using them. However, every expression entered in a MuPAD notebook is assumed to be a combination of symbolic variables unless otherwise defined. This means that you have to be especially careful when working in MuPAD notebooks, since fewer of your typographic errors cause syntax errors.

This table lists common tasks, meaning commands or functions, and how they differ in MATLAB and MuPAD syntax.

### Common Tasks in MATLAB and MuPAD Syntax

Task	MATLAB syntax	MuPAD syntax
Assignment	=	:=
List variables	whos	anames(All, User)
Numerical value of expression	double( <i>expression</i> )	float( <i>expression</i> )
Suppress output	;	:
Enter matrix	[x11,x12,x13; x21,x22,x23]	matrix([[x11,x12,x13], [x21,x22,x23]])
{a,b,c}	cell array	set
Linear algebra commands	Nothing extra needed	linalg:: prefix, or use(linalg)
Autocompletion	<b>Tab</b>	<b>Ctrl-space</b>
Equality, inequality comparison	==, ~=	=, <>

The next table lists the differences between MATLAB expressions and MuPAD expressions.

### **MATLAB vs. MuPAD Expressions**

<b>MATLAB Expression</b>	<b>MuPAD Expression</b>
Inf	infinity
pi	PI
i	I
NaN	undefined
fix	trunc
log	ln
asin	arcsin
acos	arccos
atan	arctan
asinh	arcsinh
acosh	arccosh
atanh	arctanh
acsc	arccsc
asec	arcsec
acot	arccot
acsch	arccsch
asech	arcsech
acoth	arccoth
besselj	besselJ
bessely	besselY
besseli	besselI
besselk	besselK
lambertw	lambertW
sinint	Si

### MATLAB vs. MuPAD Expressions (Continued)

MATLAB Expression	MuPAD Expression
cosint	Ci
eulergamma	EULER
conj	conjugate
catalan	CATALAN
laplace	transform::laplace
ilaplace	transform::invlaplace
ztrans	transform::ztrans
iztrans	transform::invztrans

The MuPAD definition of Fourier transform and inverse Fourier transform differ from their Symbolic Math Toolbox counterparts by the sign of the exponent:

	Symbolic Math Toolbox definition	MuPAD definition
Fourier transform	$F[f](w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx$ <p>F = fourier(f)</p>	$F[f](w) = \int_{-\infty}^{\infty} f(x)e^{iwx} dx$ <p>F = transform::fourier(f,x,w)</p>
Inverse Fourier transform	$F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{iwx} dw$ <p>Finv = ifourier(f)</p>	$F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{-iwx} dw$ <p>Finv = transform::invfourier(f,w,x)</p>

The MuPAD definition of exponential integral differs from the Symbolic Math Toolbox counterpart:



	<b>Symbolic Math Toolbox definition</b>	<b>MuPAD definition</b>
Exponential integral	$\text{expint}(x) = -\text{Ei}(-x) =$ $\int_x^{\infty} \frac{\exp(-t)}{t} dt$ for $x > 0 =$ $\text{Ei}(1, x).$	$\text{Ei}(x) = \int_{-\infty}^x \frac{e^t}{t} dt$ for $x < 0.$ $\text{Ei}(n, t) = \int_1^{\infty} \frac{\exp(-xt)}{t^n} dt.$ <p>The definitions of Ei extend to the complex plane, with a branch cut along the negative real axis.</p>

## Integration of MuPAD and MATLAB

### In this section...

“Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks” on page 4-24

“Calling MuPAD Functions at the MATLAB Command Line” on page 4-27

“Clearing Assumptions and Resetting the Symbolic Engine” on page 4-29

### Copying Variables and Expressions Between the MATLAB Workspace and MuPAD Notebooks

You can copy a variable in a MuPAD notebook to a variable in the MATLAB workspace using a MATLAB command. Similarly, you can copy a variable or symbolic expression in the MATLAB workspace to a variable in a MuPAD notebook using a MATLAB command. In order to do either assignment, you need to know the handle to the MuPAD notebook you want to address.

The only way to assign variables between a MuPAD notebook and the MATLAB workspace is to start the notebook using the syntax

```
nb = mupad;
```

(you can use any variable name for the handle *nb*) or open an existing notebook file with

```
nb = mupad(file_name);
```

This handle is used only for communication between the MATLAB workspace and the MuPAD notebook.

- To copy a symbolic variable in the MATLAB workspace to a variable in the MuPAD notebook engine with the same name, enter

```
setVar(notebook_handle,variable)
```

at the MATLAB command line. For example, if *nb* is the handle to the notebook and *z* is the variable, enter

```
setVar(nb,z)
```

There is no indication in the MuPAD notebook that the variable  $z$  exists. Check that it exists by entering  $z$  in an input area of the notebook, or by entering the command `anames(All, User)` in the notebook.

- To assign a symbolic expression to a variable in a MuPAD notebook, enter

```
setVar(notebook_handle, 'variable', expression)
```

at the MATLAB command line. For example, if `nb` is the handle to the notebook, `exp(x) - sin(x)` is the expression, and  $z$  is the variable, enter

```
syms x
setVar(nb, 'z', exp(x) - sin(x))
```

For this type of assignment,  $x$  must be a symbolic variable in the MATLAB workspace.

Again, there is no indication in the MuPAD notebook that the variable  $z$  exists. Check that it exists by entering  $z$  in an input area of the notebook, or by entering the command `anames(All, User)` in the notebook.

- To copy a symbolic variable in a MuPAD notebook to a variable in the MATLAB workspace, enter

```
MATLABvar = getVar(notebook_handle, 'variable');
```

at the MATLAB command line. For example, if `nb` is the handle to the notebook,  $z$  is the variable in the MuPAD notebook, and  $u$  is the variable in the MATLAB workspace, enter

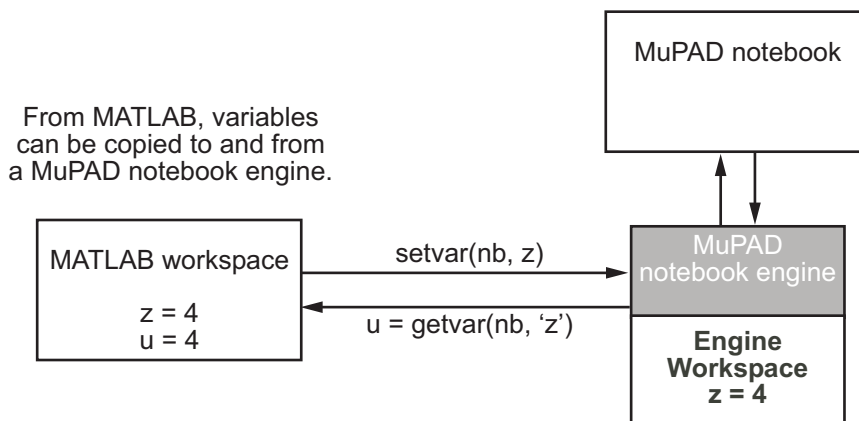
```
u = getVar(nb, 'z')
```

The communication between the MATLAB workspace and the MuPAD notebook takes place with the notebook's engine. Therefore, the variable  $z$  must be synchronized into the notebook's MuPAD engine before using `getVar`, and not merely displayed in the notebook. If you try to use `getVar` to copy a variable  $z$  that is undefined in the MuPAD engine, the resulting MATLAB variable  $u$  is empty. For more information on this topic, see "Synchronizing a Notebook and its Engine" on page 4-18.

---

**Note** All such copying and assignment must be done from the MATLAB workspace, not from a MuPAD notebook.

---



### Copying and Pasting Using the System Clipboard

You can also copy and paste between notebooks and the MATLAB workspace using standard editing commands. If you copy a result in a MuPAD notebook to the system clipboard, you may get the text associated with the expression, or a picture, depending on your operating system and application support.

For example, consider the following MuPAD expression.

```

[ y := exp(x) / (1+x^2)
  
$$\frac{e^x}{x^2+1}$$

]
  
```

Select the output with the mouse and copy it to the clipboard.

```

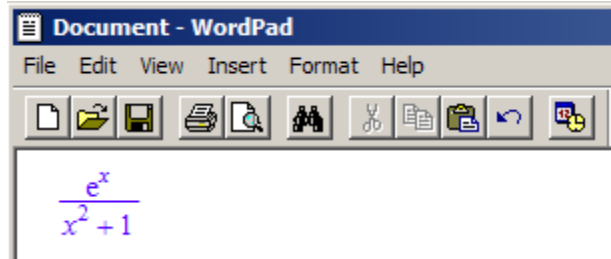
[ y := exp(x) / (1+x^2)
  
$$\frac{e^x}{x^2+1}$$

]
  
```

Paste this into the MATLAB workspace. The result is text:

$$\exp(x)/(x^2 + 1)$$

If you paste it into WordPad on a Windows® system, the result is a picture.



## Calling MuPAD Functions at the MATLAB Command Line

To access MuPAD functions and procedures at the MATLAB command line, use the `evalin(symengine, ...)` function or the `feval(symengine, ...)` function. These functions are designed to work like the existing MATLAB functions `evalin` and `feval`.

### **evalin**

For `evalin`, the syntax is

```
y = evalin(symengine, 'MuPAD_Expression');
```

Use `evalin` when you want to perform computations in the MuPAD language, while working in the MATLAB workspace. You can use `evalin` for any MuPAD expression.

For example, to make a three-element symbolic vector of the `sin(kx)` function, `k = 1 to 3`, enter

```
y = evalin(symengine, 'sin(k*x)$k=1..3')
y =
[ sin(x), sin(2*x), sin(3*x)]
```

**feval**

For evaluating a MuPAD function, you can also use the `feval` function. `feval` has a different syntax than `evalin`, so it can be simpler to use. The syntax is

```
y = feval(symengine, 'MuPAD_Function', x1, ..., xn);
```

*MuPAD\_Function* represents the name of a MuPAD function. The arguments  $x_1, \dots, x_n$  must be symbolic variables, numbers, or strings. For example, to find the tenth element in the Fibonacci sequence, enter

```
z = feval(symengine, 'numlib::fibonacci', 10)
```

```
z =  
55
```

The next example compares the use of a symbolic solution of an equation to the solution returned by the MuPAD numeric `fsolve` function near the point  $x = 3$ . (For information on this function, enter `doc(symengine, 'numeric::fsolve')` at the MATLAB command line.)

```
syms x  
f = sin(x^2);  
solve(f)  
  
ans =  
0  
0  
  
feval(symengine, 'numeric::fsolve', f, 'x=3')  
  
ans =  
x = 3.0699801238394654654386548746678
```

As you might expect, the answer is the numerical value of  $\sqrt{3\pi}$ . The setting of MATLAB format does not affect the display; it is the full returned value from the MuPAD `'numeric::fsolve'` function.

**Usage of evalin vs. feval**

The `evalin(symengine, ...)` function causes the MuPAD engine to evaluate a string. Since the MuPAD engine workspace is generally empty, expressions

returned by `evalin(symengine,...)` are not simplified or evaluated according to their definitions in the MATLAB workspace.

For example:

```
syms x
y = x^2;
evalin(symengine, 'cos(y)')

ans =
cos(y)
```

The variable `y` is not expressed in terms of `x` because `y` is not known to the MuPAD engine workspace.

In contrast, `feval(symengine,...)` can pass symbolic variables that exist in the MATLAB workspace, and these variables are evaluated before being processed in the MuPAD engine. For example:

```
syms x
y = x^2;
feval(symengine, 'cos', y)

ans =
cos(x^2)
```

## Clearing Assumptions and Resetting the Symbolic Engine

The symbolic engine workspace associated with the MATLAB workspace is usually empty. The MATLAB workspace keeps track of the values of symbolic variables, and passes them to the symbolic engine for evaluation as necessary. However, the symbolic engine workspace contains all assumptions you make about symbolic variables, such as whether a variable is real or positive. These assumptions can affect solutions to equations, simplifications, and transformations, as explained in “Examples of the Effect of Assumptions” on page 4-32.

---

**Note** The commands

```
syms x
x = sym('x')
clear x
```

clear any existing value of  $x$  in the MATLAB workspace, but do not clear assumptions about  $x$  in the symbolic engine workspace.

---

- If you make an assumption about the nature of a variable, e.g., by the commands

```
syms x real
```

or

```
syms x positive
```

clearing the variable  $x$  from the MATLAB workspace does not clear the assumption from the symbolic engine workspace. To clear the assumption, enter the command

```
syms x clear
```

For more detail, see “Checking a Variable’s Assumptions” on page 4-31 and “Examples of the Effect of Assumptions” on page 4-32.

- If you reset the symbolic engine by entering the command

```
reset(symengine)
```

or if you change symbolic engines with the `symengine` command, MATLAB no longer recognizes any symbolic variables that exist in the MATLAB workspace. Clear the variables with the `clear` command, or renew them with the `syms` or `sym` commands. The `symengine` command is discussed in “Choosing a Maple or MuPAD Engine” on page 4-34.

Here is an example of how the MATLAB workspace and the symbolic engine workspace respond to a sequence of commands:



Step	Command	MATLAB workspace	MuPAD Engine Workspace
1	<code>syms x positive</code>	x	x is positive
2	<code>clear x</code>	empty	x is positive
3	<code>syms x</code>	x	x is positive
4	<code>syms x clear</code>	x	empty

### Checking a Variable's Assumptions

To check whether a variable, say  $x$ , has any assumption in the symbolic engine workspace associated with the MATLAB workspace, enter the command

```
evalin(symengine, 'getprop(x)')
```

at the MATLAB command line.

- If the returned answer is `C_`, there are no assumptions about the variable. (`C_` means it can be any complex number.)
- If the returned value is anything else, there are assumptions about the variable.

For example:

```
syms x real
evalin(symengine, 'getprop(x)')

ans =
R_

syms z
evalin(symengine, 'assume(z <> 0)')
evalin(symengine, 'getprop(z)')

ans =
C_ minus {0}

syms z clear
evalin(symengine, 'getprop(z)')
```

```
ans =
C_
```

For more information about the basic sets that can be returned as assumptions, enter

```
doc(symengine, 'solvelib::BasicSet')
```

### Examples of the Effect of Assumptions

Assumptions can change the answers that the `solve` function returns, and can change the results of simplifications. The only assumptions you can make using MATLAB commands are `real` or `positive`.

For example, consider what transpires when solving the equation  $x^3 = 1$ :

```
syms x
solve('x^3=1')

ans =
      1
- (3^(1/2)*i)/2 - 1/2
 (3^(1/2)*i)/2 - 1/2
```

```
syms x real
solve('x^3=1')

ans =
1
```

However, clearing `x` does not change the underlying assumption that `x` is real:

```
clear x
syms x
solve('x^3=1')

ans =
1
```

Clearing `x` with `syms x clear` clears the assumption:

```
syms x clear
solve('x^3=1')

ans =
      1
- (3^(1/2)*i)/2 - 1/2
 (3^(1/2)*i)/2 - 1/2
```

Using `evalin` or `feval`, you can make a variety of assumptions about an expression; see “Calling MuPAD Functions at the MATLAB Command Line” on page 4-27. All such assumptions can be cleared with the command `syms x clear`, as in this example:

```
evalin(symengine, 'assume(a <> 0)');
evalin(symengine, 'solve(a*x^2+b*x+c=0,x)')

ans =
{-(b - (b^2 - 4*a*c)^(1/2))/(2*a),...
 -(b + (b^2 - 4*a*c)^(1/2))/(2*a)}

syms a clear
evalin(symengine, 'solve(a*x^2+b*x+c=0,x)')

ans =
piecewise([a <> 0, {-(b - (b^2 - 4*a*c)^(1/2))/(2*a),...
 -(b + (b^2 - 4*a*c)^(1/2))/(2*a)}],...
 [a = 0 and b <> 0, {-c/b}], [a = 0 and b = 0 and c = 0, C_],...
 [a = 0 and b = 0 and c <> 0, {}])
```

## Using Maple and MuPAD Engines

### In this section...

“Choosing a Maple or MuPAD Engine” on page 4-34

“Differences in Maple and MuPAD Syntax” on page 4-35

“Differences in Functionality When Using MuPAD and Maple Engines” on page 4-43

### Choosing a Maple or MuPAD Engine

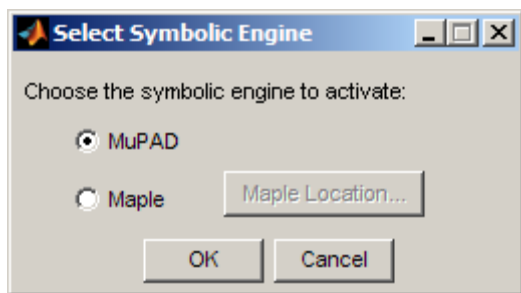
You can use a Maple engine with Symbolic Math Toolbox software instead of the default MuPAD engine. You must have a compatible version of Maple software, available from Maplesoft®, installed on your computer. Contact <http://www.maplesoft.com/> to find which versions of Maple software work with Symbolic Math Toolbox software.

To choose a Maple engine:

- 1 Enter

```
symengine
```

at the MATLAB command line to display the following GUI.



- 2 • To use a Maple engine, click the **Maple** button, then the **Maple Location** button, and navigate to your computer's Maple installation directory.

To determine this directory:

- a Launch or switch to Maple.
- b Enter the command

```
kernelopts(mapledir);
```

at the Maple prompt.

- To use a MuPAD engine, click the **MuPAD** button.
- 3 Clear all symbolic variables if you change engines while using Symbolic Math Toolbox software. Do this by entering the `clear` command, or `syms variable-list`.

---

**Caution** Results may differ when computed with a Maple engine compared to those computed with a MuPAD engine. Also, the capabilities of the engines may differ.

---

## Differences in Maple and MuPAD Syntax

Prior to Version 5 of Symbolic Math Toolbox, Extended Symbolic Math Toolbox™ software supported calling the Maple engine from the MATLAB workspace with the `maple` and `procread` functions. Symbolic Math Toolbox software now uses the MuPAD engine by default, and uses the `evalin(symengine,...)` and `feval(symengine,...)` functions to access the MuPAD engine from the MATLAB workspace.

---

**Caution** Be sure to select the functions and syntax that are appropriate for your selected symbolic engine. For example, use `procread` and `maple` with a Maple engine; use `evalin` and `feval` with a MuPAD engine. The tables in the remainder of this section show the differences in syntax between the two engines.

---

## The `maple` Function and MuPAD Calls

The `maple` function is equivalent to one of three MuPAD calls, depending on its syntax:

Maple Engine Syntax	MuPAD Engine Syntax
<code>maple(command)</code>	<code>evalin(symengine, 'command')</code>
<code>maple('function', arg1, arg2, ...)</code>	<code>feval(symengine, 'function', arg1, arg2, ...)</code>
<code>maple restart</code>	<code>reset(symengine)</code>

- The content of the command and function names must be in the correct syntax, as well as the MATLAB function call. The tables in “Maple and MuPAD Syntax” on page 4-38, “Maple and MuPAD Library Calls” on page 4-39, and “Maple and MuPAD Constants” on page 4-42 describe the differences between Maple and MuPAD syntax.
- The `feval` and `evalin` functions return symbolic expressions, while the `maple` command returns strings or symbolic expressions, depending on the types of the inputs.
- To convert a symbolic expressions to a string, call the `char` method. For example, if `x` is a symbolic variable equal to  $z^2/\cos(z)$ , then

```
y = char(x)
```

sets `y` to the string `'z^2/cos(z)'`.

### The procread Function and evalin Calls

The `procread` function, used with a Maple engine, is equivalent to an appropriate `evalin(symengine, ...)` method for a MuPAD engine. The `evalin` call should contain a command to read the code, as shown in the table below.

The following table contains an example in both Maple and MuPAD syntax.

<b>Example of procread and evalin Calls</b>	
<b>procread for Maple Engine</b>	<b>evalin for MuPAD Engine</b>
<pre>procread('check.src')</pre> <p>The check.src file contains the following code in Maple syntax:</p> <pre>check := proc(A)   # check(A) computes A*inverse(A)   local X;   X := inverse(A);   evalm(A &amp;* X); end;</pre>	<pre>evalin(symengine, 'read("check.mu")')</pre> <p>Note that the " in the evalin call are double-quote characters, not two single-quote characters.</p> <p>The check.mu file contains the following code in MuPAD syntax:</p> <pre>check := proc(A)   // check(A) computes A*inverse(A)   local X; begin   X := 1/A;   A * X; end_proc:</pre>

### **mhhelp Calls and the doc(symengine) method**

The Maple engine mhhelp function displays help about Maple functions to the command line. The equivalent MuPAD function is the doc method of the object returned by symengine. The doc method opens MuPAD Help to the specified function. This table shows the differences in syntax.

<b>Maple Syntax</b>	<b>MuPAD Syntax</b>
mhhelp int	doc(symengine, 'int')
mhhelp convert	doc(symengine, 'rewrite')

The table uses the equivalence of the Maple function convert and the MuPAD function rewrite. To find the differing syntax between the two engines, see “Maple and MuPAD Syntax” on page 4-38 and “Maple and MuPAD Library Calls” on page 4-39.

### Maple and MuPAD Syntax

The following table lists common tasks (meaning functions or procedures) that have differences between Maple and MuPAD syntax. Blank entries in the table mean the task is not explicitly supported. Optional arguments are enclosed in angle brackets < >.

Task	Maple Syntax	MuPAD Syntax
Comment	#comment	// one-line comment /* multi-line comment */
Previous results	%, %, %%%	%, %1, %2
Timing	s:=time(); cmd1;...;cmdN; time()-s	time(), time(cmd1,...,cmdN)
Matrices	<1,2;3,4>	matrix([[1,2],[3,4]])
Appending	[op(x), a, b, ...]	append(x, a, b, ...)
Concatenating lists	[op(x),op(y)]	x.y, [op(x),op(y)]
Sequences	i^2\$i=1..5 or seq(i^2,i=1..5)	i^2\$i=1..5
	seq(f, i = m..n, e)	f(i) \$ i = m..n step e
Types	whattype(expr)	type(expr), domtype(expr)
Testing types	type(expr,type)	testtype(expr,type)
Operand queries	op(n,expr), n >= 0	op(expr,n), n >= 0
Clearing variables	x := 'x'	delete x
Namespaces	x[y], :-	x::y
Employing package	with	use
String/name concatenation	x    10	x . 10



<b>Task</b>	<b>Maple Syntax</b>	<b>MuPAD Syntax</b>
For loops	for i from 6 by 2 to 100 do body end do;	for i from 6 to 100 step 2 do body end_for
	for x in obj do body; end do	for x in obj do body; end_for
While loops	while i < 100 do i := i+10; end do	while i < 100 do i := i+10; end_while
Repeat		repeat body until cond end_repeat
If/then/else	if b then x end if	if b then x end_if
Trapping errors	traperror(stmt)	traperror(stmt, <T>, <MaxSteps>)
Try/catch	try stmt; catch string:stmt; finally stmt end	
Procedure definition	f:=proc(x) local n; body; end;	f:=proc(x) local n; begin body; end_proc;
Non-commuting multiplication	&* (& is neutral-op) or A . B	domain overload of *

### Maple and MuPAD Library Calls

The following table lists tasks that require different library function calls in Maple syntax and MuPAD syntax. Blank entries in the table mean the call is not explicitly supported. Optional arguments are enclosed in angle brackets < >.

<b>Task</b>	<b>Maple Syntax</b>	<b>MuPAD Syntax</b>
Reading data	read(filename)	read(filename, <Quiet>, <Plain>)
Opening files	open(name,mode)	fopen(filename   TempFile,<Read   Write   Append>, <Bin   Text   Raw>)

<b>Task</b>	<b>Maple Syntax</b>	<b>MuPAD Syntax</b>
Displaying results	<code>print(e1, e2, ... )</code>	<code>print(&lt;Unquoted&gt;, &lt;NoNL&gt;, &lt;KeepOrder&gt;, &lt;Typeset&gt;, object1, object2, ...)</code>
Substituting	<code>subs(x=a,f)</code>	<code>subs(f, x=a, &lt;unsimplified&gt;)</code>
Limits	<code>limit(f, x=a, &lt;left   right   real   complex&gt;)</code>	<code>limit(f, x=a, &lt;Left   Right   Real&gt;, &lt;Intervals&gt;)</code>
Mapping	<code>map(f, expr, arg1, arg2, ...)</code>	<code>map(expr, f, &lt;arg1, arg2, ...&gt;, &lt;Unsimplified&gt;)</code>
Logarithms	<code>log(x)</code>	<code>ln(x)</code>
Some special functions	<code>AiryAi(n,z)</code>	<code>airyAi(z,n)</code>
	<code>argument(z)</code>	<code>arg(z)</code>
	<code>dirac(n,x), dirac([n1..nk],[x1..xk])</code>	<code>dirac(x,n)</code>
	<code>EllipticK(z)</code> different norm	<code>EllipticK(z)</code>
	<code>EllipticPi(z,nu,k)</code> different norm	<code>EllipticPi(m,phi,n)</code>
	<code>FresnelC(x)</code> different norm	<code>fresnelC(x)</code>
	<code>GAMMA(x)</code>	<code>gamma(x)</code>
	<code>GAMMA(a,x)</code>	<code>igamma(a,x)</code>
	<code>lnGAMMA(x)</code>	<code>lngamma(x)</code>
	<code>Heaviside(t)</code>	<code>heaviside(x)</code>
	<code>JacobiAM(z,k)</code> different norm	<code>jacobiAM(u,m)</code>
	<code>Tensor[KroneckerDelta](m,n)</code>	<code>kroneckerDelta(m,n)</code>
	<code>Psi(n,x)</code>	<code>psi(x,n)</code>
<code>Wrightomega(x)</code>	<code>wrightOmega(x)</code>	

<b>Task</b>	<b>Maple Syntax</b>	<b>MuPAD Syntax</b>
Factorials	<code>n!</code> or <code>factorial(n)</code>	<code>n!</code> or <code>fact(n)</code>
Integrals	<code>int(f, x = a..b)</code>	<code>int(f, x = a..b, &lt;PrincipalValue&gt;)</code>
Taylor series	<code>taylor(f, x=a, n)</code>	<code>taylor(f, x = x0, &lt;order&gt;, &lt;mode&gt;, &lt;NoWarning&gt;)</code>
Coefficients	<code>coeffs(p, x, 't'), coeff(p, x, n)</code>	<code>coeffs(f, &lt;vars&gt;, &lt;x&gt;, n)</code>
Simplifying basic	<code>simplify(f)</code>	<code>Simplify(f, &lt;Steps&gt;), simplify(f)</code>
Simplifying to a target	<code>simplify(f, &lt;symbolic&gt;, &lt;assume=prop&gt;, n1, n2, ...)</code>	<code>simplify(f, target), target = cos, sin, exp, ln, sqrt, unit, logic, condition, or relation</code>
Simplifying special	<code>simplify[radical]</code>	<code>radsimp</code>
Combining	<code>combine(f, n, opt1, opt2, ...)</code>	<code>combine(f, &lt;[target1, target2, ...]&gt;)</code>
Rewriting	<code>convert(expr, form)</code>	<code>rewrite(expr, func)</code>
Collecting	<code>collect(f, x, &lt;form&gt;, &lt;func&gt;)</code>	<code>collect(f, x, &lt;f&gt;)</code>
Factoring	<code>factor(a, K)</code>	<code>factor(f, &lt;Adjoin = adjoin&gt;, &lt;MaxDegree = n&gt;)</code>
Finding free variables (indeterminates)	<code>indets(expr, &lt;typename&gt;)</code>	<code>freeIndets(expr)</code>
Solving equations	<code>solve(equations, variables)</code>	<code>solve(system, vars, &lt;options&gt;)</code>
Solving ODEs	<code>dsolve({ODE, ICs}, y(x), options)</code>	<code>solve(ODE)</code>
Solving recurrences	<code>rsolve(eqns, fcns)</code>	<code>solve(REC)</code>
Finding discontinuities	<code>discont(f, x)</code>	<code>discont(f, x = a .. b, &lt;Undefined&gt;)</code>
Making assumptions	<code>assume(x, real)</code>	<code>assume(x, R_)</code>

Task	Maple Syntax	MuPAD Syntax
Listing properties	<code>about(x)</code>	<code>getprop(x)</code>
Evaluating numerically	<code>evalf(expr)</code>	<code>float(expr)</code>
Evaluating Boolean	<code>evalb(expr)</code>	<code>bool(expr)</code>
Evaluating matrix	<code>evalm(a+b)</code> , <code>evalm(x.b)</code>	<code>a+b</code> , <code>a*b</code> matrices <code>a,b</code>
Parsing string	<code>parse(string,options)</code>	<code>text2expr(string)</code>
Showing code	<code>print(f)</code>	<code>expose(f)</code>
Leading terms	<code>lcoeff</code>	<code>lcoeff(p)</code> , <code>lterm(p)</code>
Partial fractions	<code>convert(expr, parfrac)</code>	<code>partfrac(f)</code>
Plotting 2-D	<code>plot(f,x=x0..x1)</code>	<code>plot(f1,f2,..., x=x0..x1)</code> , <code>plotfunc2d(f1,f2,..., x=x0..x1)</code>
Plotting 3-D	<code>plot3d(f, x=a..b, y=c..d)</code>	<code>plotfunc3d(f1,f2,..., x=x0..x1, y=y0..y1)</code>
Pseudo division	<code>Prem(p,q)</code>	<code>pdivide(p,q)</code>
Splitting objects	<code>selectremove</code>	<code>split(obj,f)</code>

### Maple and MuPAD Constants

The following table lists the constants and global symbols that differ between Maple syntax and MuPAD syntax.

Symbol	Maple Syntax	MuPAD Syntax
Catalan number	Catalan	CATALAN
Digits of precision for floats	Digits	DIGITS
Euler's gamma constant	gamma	EULER
Library locations	libname	LIBPATH, READPATH, PACKAGEPATH

<b>Symbol</b>	<b>Maple Syntax</b>	<b>MuPAD Syntax</b>
Empty sequence	NULL	null()
Default number of terms	Order	ORDER
$\pi = 3.14159\dots$	Pi	PI
Three-state Boolean logic values	True/False/FAIL	TRUE/FALSE/UNKNOWN

## Differences in Functionality When Using MuPAD and Maple Engines

The following Symbolic Math Toolbox functions introduced in the toolbox version 4.9 require that you run a MuPAD engine:

- `matlabFunction`, which converts a symbolic expression to a function handle or a file
- `emlBlock`, which converts a symbolic expression to an Embedded MATLAB Function block



# Function Reference

---

Calculus (p. 5-2)	Perform calculus operations on symbolic expressions
Linear Algebra (p. 5-2)	Symbolic matrix manipulation
Simplification (p. 5-3)	Modify or simplify symbolic data
Solution of Equations (p. 5-4)	Solve symbolic expression
Variable Precision Arithmetic (p. 5-4)	Computing that requires exact control over numeric accuracy
Arithmetic Operations (p. 5-4)	Perform arithmetic on symbolic expressions
Special Functions (p. 5-5)	Specific mathematical applications
MuPAD (p. 5-5)	Access MuPAD
Pedagogical and Graphical Applications (p. 5-6)	Provide more information with plots and calculations
Conversions (p. 5-7)	Convert symbolic data from one data type to another
Basic Operations (p. 5-7)	Basic operations of symbolic data
Integral and Z-Transforms (p. 5-9)	Perform integral transforms and z-transforms

## Calculus

<code>diff</code>	Differentiate symbolic expression
<code>int</code>	Integrate symbolic expression
<code>jacobian</code>	Compute Jacobian matrix
<code>limit</code>	Compute limit of symbolic expression
<code>symsum</code>	Evaluate symbolic sum of series
<code>taylor</code>	Taylor series expansion

## Linear Algebra

<code>colspace</code>	Return basis for column space of matrix
<code>det</code>	Compute the determinant of symbolic matrix
<code>diag</code>	Create or extract diagonals of symbolic matrices
<code>eig</code>	Compute symbolic eigenvalues and eigenvectors
<code>expm</code>	Compute symbolic matrix exponential
<code>inv</code>	Compute symbolic matrix inverse
<code>jordan</code>	Compute Jordan canonical form of matrix
<code>null</code>	Form a basis for null space of matrix
<code>poly</code>	Compute characteristic polynomial of matrix
<code>rank</code>	Compute the rank of symbolic matrix
<code>rref</code>	Compute reduced row echelon form of matrix



svd	Compute singular value decomposition of symbolic matrix
tril	Return lower triangular part of symbolic matrix
triu	Return upper triangular part of symbolic matrix

## Simplification

coeffs	List coefficients of multivariate polynomial
collect	Collect coefficients
expand	Symbolic expansion of polynomials and elementary functions
factor	Factorization
horner	Horner nested polynomial representation
numden	Numerator and denominator
simple	Search for simplest form of symbolic expression
simplify	Symbolic simplification
subexpr	Rewrite symbolic expression in terms of common subexpressions
subs	Symbolic substitution in symbolic expression or matrix

## Solution of Equations

<code>compose</code>	Functional composition
<code>dsolve</code>	Symbolic solution of ordinary differential equations
<code>finverse</code>	Functional inverse
<code>solve</code>	Symbolic solution of algebraic equations

## Variable Precision Arithmetic

<code>digits</code>	Variable precision accuracy
<code>vpa</code>	Variable precision arithmetic

## Arithmetic Operations

<code>+</code>	Addition
<code>-</code>	Subtraction
<code>*</code>	Multiplication
<code>.*</code>	Array multiplication
<code>\</code>	Left division
<code>.\</code>	Array left division
<code>/</code>	Right division
<code>./</code>	Array right division
<code>^</code>	Matrix or scalar raised to a power
<code>.^</code>	Array raised to a power

'	Complex conjugate transpose
.'	Real transpose

## Special Functions

cosint	Cosine integral
dirac	Dirac delta
heaviside	Compute Heaviside step function
hypergeom	Generalized hypergeometric
lambertw	Lambert's $W$ function
mfun	Numeric evaluation of special mathematical function
mfunlist	List special functions for use with mfun
sinint	Sine integral
zeta	Compute the Riemann zeta function

## MuPAD

clear all	Remove items from the MATLAB workspace and reset the MuPAD engine.
doc	Get help for MuPAD functions
evalin	Evaluate MuPAD expressions
feval	Evaluate MuPAD expressions
getVar	Get variable from MuPAD notebook
mupad	Start MuPAD notebook

<code>mupadwelcome</code>	Launch MuPAD interfaces
<code>openmn</code>	Open MuPAD notebook
<code>openmu</code>	Open MuPAD program file
<code>reset</code>	Close MuPAD engine
<code>setVar</code>	Assign variable in MuPAD notebook
<code>symengine</code>	Choose symbolic engine

## **Pedagogical and Graphical Applications**

<code>ezcontour</code>	Contour plotter
<code>ezcontourf</code>	Filled contour plotter
<code>ezmesh</code>	3-D mesh plotter
<code>ezmeshc</code>	Combined mesh and contour plotter
<code>ezplot</code>	Function plotter
<code>ezplot3</code>	3-D parametric curve plotter
<code>ezpolar</code>	Polar coordinate plotter
<code>ezsurf</code>	3-D colored surface plotter
<code>ezsurfz</code>	Combined surface and contour plotter
<code>funtool</code>	Function calculator
<code>rsums</code>	Interactive evaluation of Riemann sums
<code>taylortool</code>	Taylor series calculator

## Conversions

<code>ccode</code>	C code representation of symbolic expression
<code>char</code>	Convert symbolic objects to strings
<code>double</code>	Convert symbolic matrix to MATLAB numeric form
<code>emlBlock</code>	Convert symbolic expression to an Embedded MATLAB Function block
<code>fortran</code>	Fortran representation of symbolic expression
<code>int8, int16, int32, int64</code>	Convert symbolic matrix to signed integers
<code>latex</code>	LaTeX representation of symbolic expression
<code>matlabFunction</code>	Convert symbolic expression to function handle or file
<code>poly2sym</code>	Polynomial coefficient vector to symbolic polynomial
<code>single</code>	Convert symbolic matrix to single precision
<code>sym2poly</code>	Symbolic-to-numeric polynomial conversion
<code>uint8, uint16, uint32, uint64</code>	Convert symbolic matrix to unsigned integers

## Basic Operations

<code>ceil</code>	Round symbolic matrix toward positive infinity
<code>conj</code>	Symbolic complex conjugate

<code>eq</code>	Perform symbolic equality test
<code>findsym</code>	Determine variables in symbolic expression or matrix
<code>fix</code>	Round toward zero
<code>floor</code>	Round symbolic matrix toward negative infinity
<code>frac</code>	Symbolic matrix elementwise fractional parts
<code>imag</code>	Imaginary part of complex number
<code>log10</code>	Logarithm base 10 of entries of symbolic matrix
<code>log2</code>	Logarithm base 2 of entries of symbolic matrix
<code>mod</code>	Symbolic matrix elementwise modulus
<code>pretty</code>	Pretty-print symbolic expressions
<code>quorem</code>	Symbolic matrix elementwise quotient and remainder
<code>real</code>	Real part of a complex symbolic number
<code>round</code>	Symbolic matrix elementwise round
<code>size</code>	Symbolic matrix dimensions
<code>sort</code>	Sort symbolic vectors or polynomials
<code>sym</code>	Symbolic numbers, variables, and objects
<code>syms</code>	Shortcut for constructing symbolic objects

symvar

Find symbolic variables in symbolic expression or matrix

unique

Find unique elements of symbolic matrices or their numeric approximations

## Integral and Z-Transforms

fourier

Fourier integral transform

ifourier

Inverse Fourier integral transform

ilaplace

Inverse Laplace transform

iztrans

Inverse  $z$ -transform

laplace

Laplace transform

ztrans

$z$ -transform





# Functions — Alphabetical List

---

# Arithmetic Operations

---

<b>Purpose</b>	Perform arithmetic operations on symbols
<b>Syntax</b>	A+B A-B A*B A.*B A\B A.\B B/A A./B A^B A.^B A' A.'
<b>Description</b>	<p><b>+</b> Matrix addition. <math>A+B</math> adds <math>A</math> and <math>B</math>. <math>A</math> and <math>B</math> must have the same dimensions, unless one is scalar.</p> <p><b>-</b> Matrix subtraction. <math>A-B</math> subtracts <math>B</math> from <math>A</math>. <math>A</math> and <math>B</math> must have the same dimensions, unless one is scalar.</p> <p><b>*</b> Matrix multiplication. <math>A*B</math> is the linear algebraic product of <math>A</math> and <math>B</math>. The number of columns of <math>A</math> must equal the number of rows of <math>B</math>, unless one is a scalar.</p> <p><b>.*</b> Array multiplication. <math>A.*B</math> is the entry-by-entry product of <math>A</math> and <math>B</math>. <math>A</math> and <math>B</math> must have the same dimensions, unless one is scalar.</p> <p><b>\</b> Matrix left division. <math>A\B</math> solves the symbolic linear equations <math>A*X=B</math> for <math>X</math>. Note that <math>A\B</math> is roughly equivalent to <math>\text{inv}(A)*B</math>. Warning messages are produced if <math>X</math> does not exist or is not unique. Rectangular matrices <math>A</math> are allowed, but the equations must be consistent; a least squares solution is <i>not</i> computed.</p>

- .\      Array left division.  $A.\backslash B$  is the matrix with entries  $B(i,j)/A(i,j)$ .  $A$  and  $B$  must have the same dimensions, unless one is scalar.
- /      Matrix right division.  $B/A$  solves the symbolic linear equation  $X*A=B$  for  $X$ . Note that  $B/A$  is the same as  $(A.\backslash B.')$ '. Warning messages are produced if  $X$  does not exist or is not unique. Rectangular matrices  $A$  are allowed, but the equations must be consistent; a least squares solution is not computed.
- ./     Array right division.  $A./B$  is the matrix with entries  $A(i,j)/B(i,j)$ .  $A$  and  $B$  must have the same dimensions, unless one is scalar.
- ^      Matrix power.  $A^B$  raises the square matrix  $A$  to the integer power  $B$ . If  $A$  is a scalar and  $B$  is a square matrix,  $A^B$  raises  $A$  to the matrix power  $B$ , using eigenvalues and eigenvectors.  $A^B$ , where  $A$  and  $B$  are both matrices, is an error.
- .^     Array power.  $A.^B$  is the matrix with entries  $A(i,j)^B(i,j)$ .  $A$  and  $B$  must have the same dimensions, unless one is scalar.
- '      Matrix Hermition transpose. If  $A$  is complex,  $A'$  is the complex conjugate transpose.
- .'     Array transpose.  $A.'$  is the real transpose of  $A$ .  $A.'$  does not conjugate complex entries.

## Examples

The following statements

```
syms a b c d;  
A = [a b; c d];  
A*A/A  
A*A-A^2
```

return

# Arithmetic Operations

---

```
[ a, b]  
[ c, d]
```

```
[ 0, 0]  
[ 0, 0]
```

The following statements

```
syms a11 a12 a21 a22 b1 b2;  
A = [a11 a12; a21 a22];  
B = [b1 b2];  
X = B/A;  
x1 = X(1)  
x2 = X(2)
```

return

```
x1 =  
(a22*b1 - a21*b2)/(a11*a22 - a12*a21)  
  
x2 =  
-(a12*b1 - a11*b2)/(a11*a22 - a12*a21)
```

## See Also

null, solve

<b>Purpose</b>	C code representation of symbolic expression
<b>Syntax</b>	<pre>ccode(s) ccode(s, 'file', fileName)</pre>
<b>Description</b>	<p><code>ccode(s)</code> returns a fragment of C that evaluates the symbolic expression <code>s</code>.</p> <p><code>ccode(s, 'file', fileName)</code> writes an “optimized” C code fragment that evaluates the symbolic expression <code>s</code> to the file named <code>fileName</code>. “Optimized” means intermediate variables are automatically generated in order to simplify the code.</p>
<b>Examples</b>	<p>The statements</p> <pre>syms x f = taylor(log(1+x)); ccode(f)</pre> <p>return</p> <pre>t0 = x - (x*x)*(1.0/2.0) + (x*x*x)*(1.0/3.0) - (x*x*x*x)*(1.0/4.0) + (x*x*x*x*x)*(1.0/5.0);</pre> <p>The statements</p> <pre>H = sym(hilb(3)); ccode(H)</pre> <p>return</p> <pre>H[0][0] = 1.0; H[0][1] = 1.0/2.0; H[0][2] = 1.0/3.0; H[1][0] = 1.0/2.0; H[1][1] = 1.0/3.0; H[1][2] = 1.0/4.0;</pre>

```
H[2][0] = 1.0/3.0;  
H[2][1] = 1.0/4.0;  
H[2][2] = 1.0/5.0;
```

The statements

```
syms x  
z = exp(-exp(-x));  
ccode(diff(z,3), 'file', 'ccodetest');
```

return a file named `ccodetest` containing the following:

```
t2 = exp(-x);  
t3 = exp(-t2);  
t0 = t3*exp(x*(-2.0))*(-3.0)+t3*exp(x*(-3.0))+t2*t3;
```

## See Also

`fortran`, `latex`, `matlabFunction`, `pretty`

“Generating Code from Symbolic Expressions” on page 3-129

**Purpose** Round symbolic matrix toward positive infinity

**Syntax**  $Y = \text{ceil}(x)$

**Description**  $Y = \text{ceil}(x)$  is the matrix of the smallest integers greater than or equal to  $x$ .

**Example**

```
x = sym(-5/2);  
[fix(x) floor(x) round(x) ceil(x) frac(x)]
```

```
ans =  
[ -2, -3, -3, -2, -1/2]
```

**See Also** round, floor, fix, frac

# char

---

**Purpose** Convert symbolic objects to strings

**Syntax** `char(A)`  
`char(A, d)`

**Description** `char(A)` converts a symbolic scalar or a symbolic array to a string.  
`char(A, d)` converts a symbolic scalar or array to a string. For symbolic arrays, the second parameter specifies the form of the resulting string. For symbolic scalars, this parameter does not affect the result. `d` will be removed in a future release.

**Inputs** `A`  
A symbolic scalar or a symbolic array

`d`  
`d` will be removed in a future release.  
A number that specifies the format of the resulting string. For symbolic arrays:

`char(A, 1)` results in `matrix([...])`  
`char(A, 2)` results in `matrix([[...],[...]])`  
`char(A, d)` for all other values of the parameter `d` results in `array([1..m, 1..n, 1..p], [(1,1,1) = xxx, ..., (m,n,p) = xxx])`

**Examples** Convert symbolic expressions to strings, and then concatenate the strings:

```
syms x;  
y = char(x^3 + x^2 + 2*x - 1);  
name = [y, ' presents a polynomial expression']
```



The result is:

```
name =  
x^3 + x^2 + 2*x - 1 presents a polynomial expression
```

Convert a symbolic matrix to a string:

```
A = sym(hilb(3))  
char(A)
```

The result is:

```
A =  
[ 1, 1/2, 1/3]  
[ 1/2, 1/3, 1/4]  
[ 1/3, 1/4, 1/5]  
  
ans =  
matrix([[1,1/2,1/3],[1/2,1/3,1/4],[1/3,1/4,1/5]])
```

## See Also

[sym](#) | [double](#) | [pretty](#)

# clear all

---

**Purpose** Remove items from the MATLAB workspace and reset the MuPAD engine.

**Syntax** `clear all`

**Description** `clear all` clears all objects in the MATLAB workspace and closes the MuPAD engine associated with the MATLAB workspace resetting all its assumptions.

**See Also** `reset`

**Purpose** List coefficients of multivariate polynomial

**Syntax**

```
C = coffs(p)
C = coffs(p, x)
[C, T] = coffs(p, x)
```

**Description**

`C = coffs(p)` returns the coefficients of the polynomial `p` with respect to all the indeterminates of `p`.

`C = coffs(p, x)` returns the coefficients of the polynomial `p` with respect to `x`.

`[C, T] = coffs(p, x)` returns a list of the coefficients and a list of the terms of `p`. There is a one-to-one correspondence between the coefficients and the terms of `p`.

**Examples** List the coefficients of the following single-variable polynomial:

```
syms x
t = 16*log(x)^2 + 19*log(x) + 11;
coffs(t)
```

The result is:

```
ans =
[ 11, 19, 16]
```

List the coefficients of the following polynomial with respect to the indeterminate `sin(x)`:

```
syms a b c x
y = a + b*sin(x) + c*sin(2*x);
coffs(y, sin(x))
```

The result is:

```
ans =  
[ a + c*sin(2*x), b]
```

---

List the coefficients of the following multivariable polynomial with respect to all the indeterminates and with respect to the variable  $x$  only:

```
syms x y  
z = 3*x^2*y^2 + 5*x*y^3;  
coeffs(z)  
coeffs(z,x)
```

The results are:

```
ans =  
[ 5, 3]  
  
ans =  
[ 5*y^3, 3*y^2]
```

---

Display the list of the coefficients and the list of the terms of the following polynomial expression:

```
syms x y  
z = 3*x^2*y^2 + 5*x*y^3;  
[c,t] = coeffs(z,y)
```

The results are:

```
c =  
[ 5*x, 3*x^2]  
  
t =  
[ y^3, y^2]
```

## See Also

`sym2poly`

<b>Purpose</b>	Collect coefficients
<b>Syntax</b>	$R = \text{collect}(S)$ $R = \text{collect}(S, v)$
<b>Description</b>	$R = \text{collect}(S)$ returns an array of collected polynomials for each polynomial in the array $S$ of polynomials. $R = \text{collect}(S, v)$ collects terms containing the variable $v$ .
<b>Examples</b>	The following statements  <pre>syms x y; R1 = collect((exp(x)+x)*(x+2)) R2 = collect((x+y)*(x^2+y^2+1), y) R3 = collect([(x+1)*(y+1), x+y])</pre> return  $R1 =$ $x^2 + (\exp(x) + 2)*x + 2*\exp(x)$  $R2 =$ $y^3 + x*y^2 + (x^2 + 1)*y + x*(x^2 + 1)$  $R3 =$ $[ y + x*(y + 1) + 1, x + y ]$
<b>See Also</b>	<code>expand</code> , <code>factor</code> , <code>simple</code> , <code>simplify</code> , <code>syms</code>

# colspace

---

**Purpose** Return basis for column space of matrix

**Syntax** `B = colspace(A)`

**Description** `B = colspace(A)` returns a matrix whose columns form a basis for the column space of `A`. The matrix `A` is symbolic or numeric.

**Examples** Find the basis for the column space of the following matrix:

```
A = sym([2,0;3,4;0,5])
B = colspace(A)
```

The result is:

```
A =
[ 2, 0]
[ 3, 4]
[ 0, 5]

B =
[ 1, 0]
[ 0, 1]
[-15/8, 5/4]
```

**See Also** `null` | `size`

**Purpose**

Functional composition

**Syntax**

```
compose(f,g)
compose(f,g,z)
compose(f,g,x,z)
compose(f,g,x,y,z)
```

**Description**

`compose(f,g)` returns  $f(g(y))$  where  $f = f(x)$  and  $g = g(y)$ . Here  $x$  is the symbolic variable of  $f$  as defined by `symvar` and  $y$  is the symbolic variable of  $g$  as defined by `symvar`.

`compose(f,g,z)` returns  $f(g(z))$  where  $f = f(x)$ ,  $g = g(y)$ , and  $x$  and  $y$  are the symbolic variables of  $f$  and  $g$  as defined by `symvar`.

`compose(f,g,x,z)` returns  $f(g(z))$  and makes  $x$  the independent variable for  $f$ . That is, if  $f = \cos(x/t)$ , then `compose(f,g,x,z)` returns  $\cos(g(z)/t)$  whereas `compose(f,g,t,z)` returns  $\cos(x/g(z))$ .

`compose(f,g,x,y,z)` returns  $f(g(z))$  and makes  $x$  the independent variable for  $f$  and  $y$  the independent variable for  $g$ . For  $f = \cos(x/t)$  and  $g = \sin(y/u)$ , `compose(f,g,x,y,z)` returns  $\cos(\sin(z/u)/t)$  whereas `compose(f,g,x,u,z)` returns  $\cos(\sin(y/z)/t)$ .

**Examples**

Suppose

```
syms x y z t u;
f = 1/(1 + x^2); g = sin(y); h = x^t; p = exp(-y/u);
```

Then

```
a = compose(f,g)
b = compose(f,g,t)
c = compose(h,g,x,z)
d = compose(h,g,t,z)
e = compose(h,p,x,y,z)
f = compose(h,p,t,u,z)
```

returns:

## compose

---

```
a =  
1/(sin(y)^2 + 1)
```

```
b =  
1/(sin(t)^2 + 1)
```

```
c =  
sin(z)^t
```

```
d =  
x^sin(z)
```

```
e =  
(1/exp(z/u))^t
```

```
f =  
x^(1/exp(y/z))
```

### See Also

`finverse`, `subs`, `syms`



<b>Purpose</b>	Symbolic complex conjugate
<b>Syntax</b>	<code>conj(X)</code>
<b>Description</b>	<code>conj(X)</code> is the complex conjugate of $X$ . For a complex $X$ , $\text{conj}(X) = \text{real}(X) - i*\text{imag}(X)$ .
<b>See Also</b>	<code>real</code> , <code>imag</code>

# cosint

---

**Purpose** Cosine integral

**Syntax** `Y = cosint(X)`

**Description** `Y = cosint(X)` evaluates the cosine integral function at the elements of `X`, a numeric matrix, or a symbolic matrix. The cosine integral function is defined by

$$Ci(x) = \gamma + \ln(x) + \int_0^x \frac{\cos t - 1}{t} dt,$$

where  $\gamma$  is Euler's constant 0.577215664...

**Examples** Compute cosine integral for a numerical value:

```
cosint(7.2)
```

The result is:

```
0.0960
```

Compute the cosine integral for `[0:0.1:1]` :

```
cosint([0:0.1:1])
```

The result is:

```
Columns 1 through 6
```

```
-Inf -1.7279 -1.0422 -0.6492 -0.3788 -0.1778
```

```
Columns 7 through 11
```

```
-0.0223 0.1005 0.1983 0.2761 0.3374
```

The statements

```
syms x;  
f = cosint(x);  
diff(f)  
  
return  
  
cos(x)/x
```

**See Also**

sinint

# det

---

**Purpose** Compute the determinant of symbolic matrix

**Syntax** `r = det(A)`

**Description** `r = det(A)` computes the determinant of A, where A is a symbolic or numeric matrix. `det(A)` returns a symbolic expression for a symbolic A and a numeric value for a numeric A.

**Examples** Compute the determinant of the following symbolic matrix:

```
syms a b c d;  
det([a, b; c, d])
```

The result is:

```
ans =  
a*d - b*c
```

---

Compute the determinant of the following matrix containing the symbolic numbers:

```
A = sym([2/3 1/3; 1 1])  
r = det(A)
```

The result is:

```
A =  
[ 2/3, 1/3]  
[ 1, 1]  
  
r =  
1/3
```

**See Also** `rank` | `eig`

**Purpose**

Create or extract diagonals of symbolic matrices

**Syntax**

```
diag(A, k)
diag(A)
```

**Description**

`diag(A, k)` returns a square symbolic matrix of order  $n + \text{abs}(k)$ , with the elements of  $A$  on the  $k$ -th diagonal.  $A$  must present a row or column vector with  $n$  components. The value  $k = 0$  signifies the main diagonal. The value  $k > 0$  signifies the  $k$ -th diagonal above the main diagonal. The value  $k < 0$  signifies the  $k$ -th diagonal below the main diagonal. If  $A$  is a square symbolic matrix, `diag(A, k)` returns a column vector formed from the elements of the  $k$ -th diagonal of  $A$ .

`diag(A)`, where  $A$  is a vector with  $n$  components, returns an  $n$ -by- $n$  diagonal matrix having  $A$  as its main diagonal. If  $A$  is a square symbolic matrix, `diag(A)` returns the main diagonal of  $A$ .

**Examples**

Create a symbolic matrix with the main diagonal presented by the elements of the vector  $v$ :

```
syms a b c;
v = [a b c];
diag(v)
```

The result is:

```
ans =
[ a, 0, 0]
[ 0, b, 0]
[ 0, 0, c]
```

---

Create a symbolic matrix with the second diagonal below the main one presented by the elements of the vector  $v$ :

```
syms a b c;
```

# diag

---

```
v = [a b c];  
diag(v, -2)
```

The result is:

```
ans =  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ a, 0, 0, 0, 0]  
[ 0, b, 0, 0, 0]  
[ 0, 0, c, 0, 0]
```

---

Extract the main diagonal from a square matrix:

```
syms a b c x y z;  
A = [a, b, c; 1, 2, 3; x, y, z];  
diag(A)
```

The result is

```
ans =  
a  
2  
z
```

---

Extract the first diagonal above the main one:

```
syms a b c x y z;  
A = [a, b, c; 1, 2, 3; x, y, z];  
diag(A, 1)
```

The result is:

```
ans =  
b  
3
```

**See Also**      `tril | triu`

# diff

---

**Purpose** Differentiate symbolic expression

**Syntax**

```
diff(expr)
diff(expr, v)
diff(expr, sym('v'))
diff(expr, n)
diff(expr, v, n)
diff(expr, n, v)
```

**Description** `diff(expr)` differentiates a symbolic expression `expr` with respect to its free variable as determined by `symvar`.

`diff(expr, v)` and `diff(expr, sym('v'))` differentiate `expr` with respect to `v`.

`diff(expr, n)` differentiates `expr`  $n$  times.  $n$  is a positive integer.

`diff(expr, v, n)` and `diff(expr, n, v)` differentiate `expr` with respect to `v`  $n$  times.

**Examples** Differentiate the following single-variable expression one time:

```
syms x;
diff(sin(x^2))
```

The result is

```
ans =
2*x*cos(x^2)
```

---

Differentiate the following single-variable expression six times:

```
syms t;
diff(t^6,6)
```

The result is



---

```
ans =  
720
```

---

Differentiate the following expression with respect to t:

```
syms x t;  
diff(sin(x*t^2), t)
```

The result is

```
ans =  
2*t*x*cos(t^2*x)
```

**See Also**

[int](#) | [jacobian](#) | [symvar](#)

# digits

---

**Purpose** Variable precision accuracy

**Syntax** `digits`  
`digits(d)`  
`d = digits`

**Description** `digits` specifies the number of significant decimal digits that MuPAD software uses to do variable precision arithmetic (VPA). The default value is 32 digits.

`digits(d)` sets the current VPA accuracy to `d` digits.

`d = digits` returns the current VPA accuracy.

**Examples** If

```
z = 1.0e-16
x = 1.0e+2
digits(14)
```

then

```
y = vpa(x*z+1)
```

uses 14-digit decimal arithmetic and returns

```
y =
1.0
```

Whereas

```
digits(15)
y = vpa(x*z+1)
```

used 15-digit decimal arithmetic and returns

```
y =
1.000000000000001
```

**See Also**      `double, vpa`

# dirac

---

**Purpose** Dirac delta

**Syntax** `dirac(x)`

**Description** `dirac(x)` returns the Dirac delta function of  $x$ .

The Dirac delta function, `dirac`, has the value 0 for all  $x$  not equal to 0 and the value `Inf` for  $x = 0$ . Several Symbolic Math Toolbox functions return answers in terms of `dirac`.

**Example** `dirac` has the property that

$$\int_{-\infty}^{\infty} \text{dirac}(x-a) * f(x) = f(a)$$

for any function  $f$  and real number  $a$ . For example:

```
syms x a
a = 5;
int(dirac(x-a)*sin(x),-inf, inf)

ans =
sin(5)
```

`dirac` also has the following relationship to the function `heaviside`:

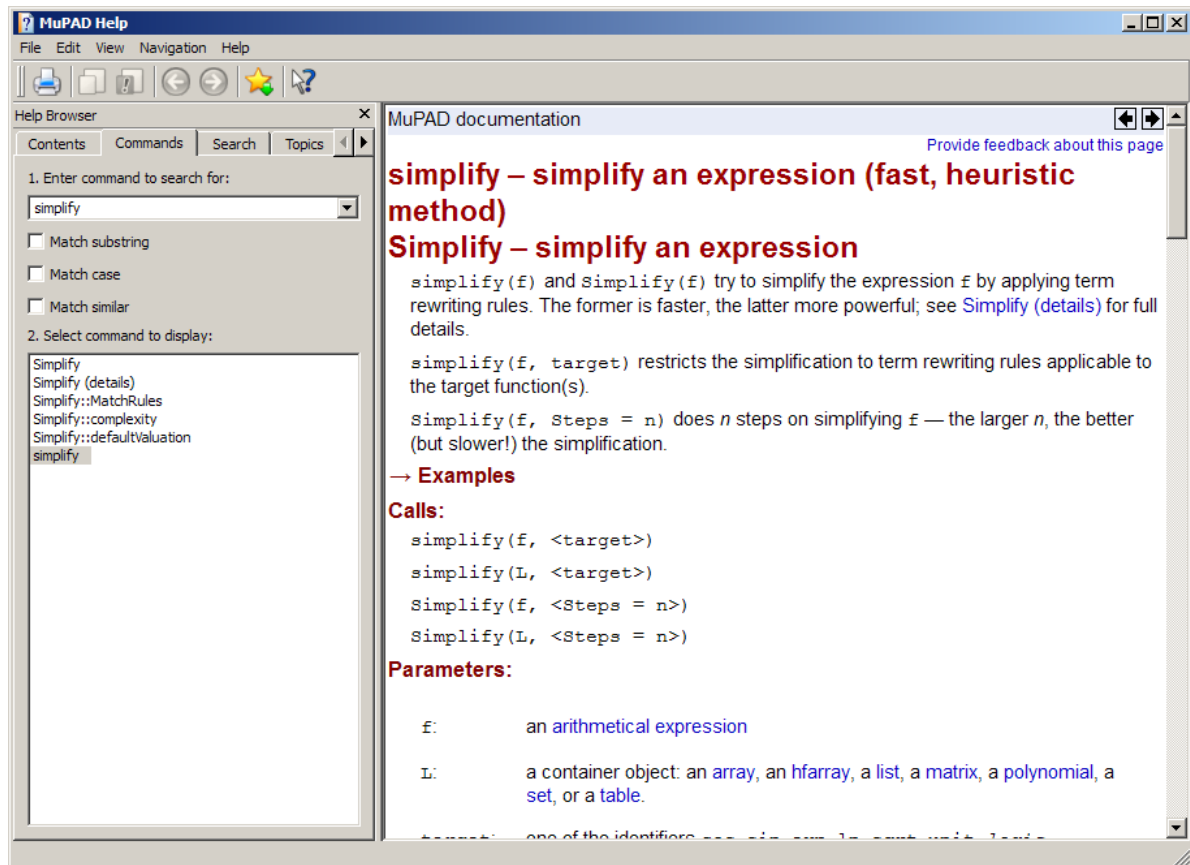
```
syms x;
diff(heaviside(x),x)

ans =
dirac(x)
```

**See Also** `heaviside`

---

<b>Purpose</b>	Get help for MuPAD functions
<b>Syntax</b>	<code>doc(symengine)</code> <code>doc(symengine, 'MuPAD_function_name')</code>
<b>Description</b>	<code>doc(symengine)</code> brings up the MuPAD help browser. <code>doc(symengine, 'MuPAD_function_name')</code> brings up the MuPAD help browser at the definition of <code>MuPAD_function_name</code> .
<b>Example</b>	<code>doc(symengine, 'simplify')</code> brings up the following window.

**See Also**

“Getting Help for MuPAD” on page 4-10

**Purpose** Convert symbolic matrix to MATLAB numeric form

**Syntax** `r = double(S)`

**Description** `r = double(S)` converts the symbolic object `S` to a numeric object. If `S` is a symbolic constant or constant expression, `double` returns a double-precision floating-point number representing the value of `S`. If `S` is a symbolic matrix whose entries are constants or constant expressions, `double` returns a matrix of double precision floating-point numbers representing the values of `S`'s entries.

**Examples** Find the numeric value for the expression  $\frac{1+\sqrt{5}}{2}$ :

```
double(sym('(1+sqrt(5))/2'))
```

The result is:

```
1.6180
```

The following statements

```
a = sym(2*sqrt(2));  
b = sym((1-sqrt(3))^2);  
T = [a, b];  
double(T)
```

```
return
```

```
ans =  
2.8284    0.5359
```

**See Also** `sym`, `vpa`

# dsolve

---

**Purpose** Symbolic solution of ordinary differential equations

**Syntax** `dsolve('eq1','eq2',...,'cond1','cond2',...,'v')`  
`dsolve(...,'IgnoreAnalyticConstraints',value)`

**Description** `dsolve('eq1','eq2',...,'cond1','cond2',...,'v')` symbolically solves the ordinary differential equations `eq1`, `eq2`, ... using `v` as the independent variable. Here `cond1`, `cond2`, ... specify boundary or initial conditions or both. You also can use the following syntax: `dsolve('eq1','eq2',...,'cond1','cond2',...,'v')`. The default independent variable is `t`.

The letter `D` denotes differentiation with respect to the independent variable. The primary default is  $d/dx$ . The letter `D` followed by a digit denotes repeated differentiation. For example, `D2` is  $d^2/dx^2$ . Any character immediately following a differentiation operator is a dependent variable. For example, `D3y` denotes the third derivative of  $y(x)$  or  $y(t)$ .

You can specify initial and boundary conditions by equations like  $y(a) = b$  or  $Dy(a) = b$ , where  $y$  is a dependent variable and  $a$  and  $b$  are constants. If the number of the specified initial conditions is less than the number of dependent variables, the resulting solutions contain the arbitrary constants `C1`, `C2`, ...

You can input each equation or a condition as a separate symbolic equation. The `dsolve` command accepts up to 12 input arguments.

`dsolve` can produce the following three types of outputs:

- For one equation and one output, `dsolve` returns the resulting solution with multiple solutions to a nonlinear equation in a symbolic vector.
- For several equations and an equal number of outputs, `dsolve` sorts the results alphabetically and assigns them to the outputs.
- For several equations and a single output, `dsolve` returns a structure containing the solutions.



If `dsolve` cannot find a closed-form (explicit) solution, it attempts to find an implicit solution. When `dsolve` returns an implicit solution, it issues a warning. If `dsolve` cannot find either an explicit or an implicit solution, then it issues a warning and returns the empty `sym`. In such a case, you can find a numeric solution, using the MATLAB `ode23` or `ode45` functions. In some cases involving nonlinear equations, the output is an equivalent lower order differential equation or an integral.

`dsolve(..., 'IgnoreAnalyticConstraints', value)` accepts the following values:

- `value = 'all'` applies the purely algebraic simplifications to the expressions on both sides of equations. These simplifications might not be generally valid. The default value of this option is `all`.
- `value = 'none'` solves ordinary differential equations without additional assumptions. The results obtained with this option are correct for all values of the arguments.

---

**Note** By default, the solver does not guarantee general correctness and completeness of the results. If you do not set the option `IgnoreAnalyticConstraints` to `none`, always verify results returned by the `dsolve` command.

---

If you do not set the value of the option `IgnoreAnalyticConstraints` to `none`, the solver applies the following rules to the expressions on both sides of an equation:

- The solutions of polynomial equations must be complete.
- $\ln(a) + \ln(b) = \ln(a \cdot b)$  for all values of  $a$  and  $b$ . In particular, the following equality is valid for all values of  $a$ ,  $b$ , and  $c$ :

$$(a \cdot b)^c = a^c \cdot b^c$$

- $\ln(a^b) = b \cdot \ln a$  for all values of  $a$  and  $b$ . In particular, the following equality is valid for all values of  $a$ ,  $b$ , and  $c$ :

$$(a^b)^c = a^{b \cdot c}$$

- The following equality is valid for all values of  $x$ :
  - $\ln(e^x) = x$
  - $\arcsin(\sin(x)) = x$ ,  $\arccos(\cos(x)) = x$ ,  $\arctan(\tan(x)) = x$
  - $\operatorname{arcsinh}(\sinh(x)) = x$ ,  $\operatorname{arccosh}(\cosh(x)) = x$ ,  $\operatorname{arctanh}(\tanh(x)) = x$
  - $W_k(x \cdot e^x) = x$  for all values of  $k$
- The solver can multiply both sides of an equation by any expression except 0.

## Examples

### Solving Ordinary Differential Equations Symbolically

```
dsolve('Dx = -a*x')
```

```
ans =  
C2/exp(a*t)
```

### Specifying the Dependent Variable

The following differential equation presents  $f$  as a dependent variable:

```
dsolve('Df = f + sin(t)')
```

```
ans =  
C4*exp(t) - sin(t)/2 - cos(t)/2
```

**Specifying the Independent Variable**

```
dsolve('(Dy)^2 + y^2 = 1', 's')
```

```
ans =
```

```

1
-1
(4*tan(C11/4 + s/4)*(tan(C11/4 + s/4)^2 - 1))/(tan(C11/4 + s/4)^2 + 1)^2
(4*tan(C7/4 - s/4)*(tan(C7/4 - s/4)^2 - 1))/(tan(C7/4 - s/4)^2 + 1)^2
```

**Setting Initial and Boundary Conditions**

```
dsolve('Dy = a*y', 'y(0) = b')
```

```
ans =
b*exp(a*t)
```

```
dsolve('D2y = -a^2*y', 'y(0) = 1', 'Dy(pi/a) = 0')
```

```
ans =
exp(a*i*t)/2 + 1/(2*exp(a*i*t))
```

**Solving a System of Differential Equations**

```
z = dsolve('Dx = y', 'Dy = -x')
```

```
z =
x: [1x1 sym]
y: [1x1 sym]
```

Enter z.x and z.y to see the results:

```
z.x
```

```
ans =
C20*cos(t) + C19*sin(t)
```

```
z.y
```

```
ans =  
C19*cos(t) - C20*sin(t)
```

## Using the IgnoreAnalyticConstraints Option

By default, the solver applies the set of purely algebraic simplifications that are not correct in general, but that can result in simple and practical solutions:

```
y = dsolve('Dy=1+y^2', 'y(0)=1')  
  
y =  
tan(pi/4 + t)
```

To obtain complete and generally correct solutions, set the value of the option `IgnoreAnalyticConstraints` to `none`:

```
y = dsolve('Dy=1+y^2', 'y(0)=1', ...  
          'IgnoreAnalyticConstraints', 'none')  
  
y =  
piecewise([C29 in Z_, tan(pi/4 + t + pi*C29)])
```

The algebraic simplifications also allow you to obtain solutions for the equations that the solver cannot compute when it uses strict mathematical rules:

```
dsolve('Dv=19.6-0.00196*v^2', 'v(0)=0')  
  
ans =  
-(100/exp((49*t)/125) - 100)/(1/exp((49*t)/125) + 1)
```

versus

```
dsolve('Dv=19.6-0.00196*v^2', 'v(0)=0', ...  
          'IgnoreAnalyticConstraints', 'none')
```

```
Warning: Explicit solution could not be found.  
> In dsolve at 104
```

```
ans =  
[ empty sym ]
```

**Diagnostics**

If `dsolve` cannot find an analytic solution for an equation, it prints the warning:

```
Warning: Explicit solution could not be found.
```

and returns an empty sym object.

**See Also**

`syms`

**Purpose** Compute symbolic eigenvalues and eigenvectors

**Syntax**

```
lambda = eig(A)
[V,D] = eig(A)
[V,D,P] = eig(A)
lambda = eig(vpa(A))
[V,D] = eig(vpa(A))
```

**Description** `lambda = eig(A)` returns a symbolic vector containing the eigenvalues of the square symbolic matrix `A`.

`[V,D] = eig(A)` returns matrices `V` and `D`. The columns of `V` present eigenvectors of `A`. The diagonal matrix `D` contains eigenvalues. If the resulting `V` has the same size as `A`, the matrix `A` has a full set of linearly independent eigenvectors that satisfy  $A*V = V*D$ .

`[V,D,P] = eig(A)` returns a vector of indices `P`. The length of `P` equals to the total number of linearly independent eigenvectors, so that  $A*V = V*D(P,P)$ .

`lambda = eig(vpa(A))` returns numeric eigenvalues using variable precision arithmetic.

`[V,D] = eig(vpa(A))` returns numeric eigenvectors using variable precision arithmetic. If `A` does not have a full set of eigenvectors, the columns of `V` are not linearly independent.

**Examples** Compute the eigenvalues for the magic square of order 5:

```
M = sym(magic(5));
eig(M)
```

The result is:

```
ans =
                                     65
(625/2 - (5*3145^(1/2))/2)^(1/2)
((5*3145^(1/2))/2 + 625/2)^(1/2)
```

$$-(625/2 - 5/2*3145^{(1/2)})^{(1/2)}$$

$$-(5/2*3145^{(1/2)} + 625/2)^{(1/2)}$$


---

Compute the eigenvalues for the magic square of order 5 using variable precision arithmetic:

```
M = sym(magic(5));
eig(vpa(M))
```

The result is:

```
ans =
                                     65.0
21.27676547147379553062642669797423
13.12628093070921880252564308594914
-13.126280930709218802525643085949
-21.276765471473795530626426697974
```

---

Compute the eigenvalues and eigenvectors for one of the MATLAB test matrices:

```
A = sym(gallery(5))
[v, lambda] = eig(A)
```

The results are:

```
A =
[ -9,    11,   -21,    63,   -252]
[  70,   -69,   141,  -421,  1684]
[ -575,   575, -1149,  3451, -13801]
[ 3891, -3891,  7782, -23345,  93365]
[ 1024, -1024,  2048, -6144,  24572]
```

```
v =
      0
21/256
```

```
-71/128  
973/256  
1
```

```
lambda =  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]  
[ 0, 0, 0, 0, 0]
```

## See Also

[jordan](#) | [poly](#) | [svd](#) | [vpa](#)



**Purpose** Convert symbolic expression to an Embedded MATLAB Function block

**Syntax**

```
emlBlock(block, f)
emlBlock(block, f1, f2, ...)
emlBlock(block, f1, f2, ..., param1, value1,...)
```

**Description** `emlBlock(block, f)` converts the symbolic expression `f` to an Embedded MATLAB Function block that you can use in Simulink models. The parameter `block` specifies the name of the block you create or modify. The `block` should be a string.

`emlBlock(block, f1, f2, ...)` converts a list of the symbolic expressions `f1, f2, ...` to an Embedded MATLAB Function block with multiple outputs.

`emlBlock(block, f1, f2, ..., param1, value1,...)` converts a list of the symbolic expressions `f1, f2, ...` to an Embedded MATLAB Function block with multiple outputs, with the following options for parameter/value pairs:

- `Parameter = 'functionName'` allows you to set the name of the function. `value` should be a string. By default, `value` coincides with the name of the block.
- `Parameter = 'outputs'` allows you to set the names of the output ports. `value` should be a cell array of strings. The number of `value` entries should equal or exceed the number of free variables in the symbolic expression `f`. The default name of an output port consists of the word `out` followed by the output port number, for example, `out3`.
- `Parameter = 'vars'` allows you to set the order of the variables and the corresponding input ports of a block. The default order is alphabetical. `value` should be either a cell array of strings or symbolic arrays, or a vector of symbolic variables. The number of `value` entries should equal or exceed the number of free variables in the symbolic expression `f`.

---

**Note** To convert a MuPAD expression or function to an Embedded MATLAB Function block, use `f = evalin(symengine, 'MuPAD_Expression')` or `f = feval(symengine, 'MuPAD_Function', x1, ..., xn)`. `emlBlock` cannot correctly convert some MuPAD expressions to a block. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, you can:

- Run the simulation containing the resulting block.
  - Open the block and verify that all the functions are defined in the Embedded MATLAB Function Library.
- 

Symbolic Math Toolbox with a Maple engine does not support `emlBlock`. For details, see “Differences in Functionality When Using MuPAD and Maple Engines” on page 4-43.

## Examples

Before you can convert a symbolic expression to an Embedded MATLAB Function block, create an empty model or open an existing one:

```
new_system('new_system');
open_system('new_system');
```

Use `emlBlock` to create the block `new_block` containing the symbolic expression:

```
syms x y z
f = x^2 + y^2 + z^2;
emlBlock('new_system/new_block', f);
```

If you use the name of an existing block, the `emlBlock` command replaces the definition of an existing block with the converted symbolic expression.

You can open and edit the resulting block:

```
function out1 = new_block(x,y,z)
%NEW_BLOCK
%   OUT1 = NEW_BLOCK(X,Y,Z)

%   This function was generated by
%   the Symbolic Math Toolbox version 5.2.
%   31-Oct-2008 11:47:28

out1 = x.^2 + y.^2 + z.^2;
```

The following example generates a block and sets the function name to *new\_function*:

```
emlBlock('new_system/new_block', x, y, z,...
'functionName', 'new_function')
```

You can change the order of the input ports:

```
emlBlock('new_system/new_block', x, y, z,...
'vars', [y z x])
```

Also, you can rename the output variables and the corresponding ports:

```
emlBlock('new_system/new_block', x, y, z,...
'outputs',{'name1','name2','name3'})
```

`emlBlock` accepts several options simultaneously:

```
emlBlock('new_system/new_block', x, y, z,...
'functionName', 'new_function','vars', [y z x],...
'outputs',{'name1','name2','name3'})
```

You also can convert MuPAD expressions:

# emlBlock

---

```
syms x y;  
f = evalin(symengine, 'arcsin(x) + arccos(y)');  
emlBlock('new_system/new_block', f);
```

The resulting block contains the same expressions written in the MATLAB language:

```
function out1 = new_block(x,y)  
%NEW_BLOCK  
%   OUT1 = NEW_BLOCK(X,Y)  
  
%   This function was generated by  
%   the Symbolic Math Toolbox version 5.2.  
%   31-Oct-2008 11:48:48  
  
out1 = asin(x) + acos(y);
```

## See Also

`ccode`, `fortran`, `matlabFunction`, `subs`, `sym2poly`

“Generating Code from Symbolic Expressions” on page 3-129

**Purpose** Perform symbolic equality test

**Syntax** `eq(A, B)`  
`A == B`

**Description** `eq(A, B)` compares each element of `A` for equality with the corresponding element of `B`. If the elements are not equal or if either element is undefined, the test fails. `eq` does not expand or simplify expressions before making the comparison.

`A == B` is the alternate syntax for `eq(A, B)`.

**Examples** Check equality of two symbolic matrices:

```
A = sym(hilb(10));
B = sym([1/11 1/12 1/13 1/14 1/15 1/16]);
eq(A(9, 3:8), B)
```

The result is:

```
ans =
     1     1     1     1     1     1
```

Check the trigonometric identity:

```
syms x;
sin(x)^2 + cos(x)^2 == 1
```

The symbolic equality test might fail to recognize mathematical equivalence of polynomial or trigonometric expressions because it does not simplify or expand them. The result is:

```
ans =
     0
```

When testing mathematical equivalence of such expressions, simplify the difference between the expressions, and then compare the result with 0:

```
syms x;  
simplify(sin(x)^2 + cos(x)^2 - 1) == 0
```

The result is:

```
ans =  
    1
```

**See Also**

`simplify`

**Purpose** Evaluate MuPAD expressions

**Syntax**

```
result = evalin(symengine, 'MuPAD_expression')  
[result,status] = evalin(symengine, 'MuPAD_expression')
```

**Description** `result = evalin(symengine, 'MuPAD_expression')` evaluates the MuPAD expression `MuPAD_expression`, and returns `result` as a symbolic object.

`[result,status] = evalin(symengine, 'MuPAD_expression')` returns the error status in `status` and the error message in `result` if `status` is nonzero. If `status` is 0, `result` is a symbolic object; otherwise, it is a string.

### Example

```
evalin(symengine, 'polylib::discrim(a*x^2+b*x+c,x)')
```

```
ans =  
b^2 - 4*a*c
```

**See Also** `doc`, `feval`

“Calling MuPAD Functions at the MATLAB Command Line” on page 4-27

# expm

---

**Purpose** Compute symbolic matrix exponential

**Syntax** `expm(A)`

**Description** `expm(A)` computes the matrix exponential of the symbolic matrix A.

**Examples** Compute the matrix exponential for the following matrix and simplify the result:

```
syms t;
A = [0 1; -1 0];
simplify(expm(t*A))
```

The result is:

```
ans =
[ cos(t), sin(t)]
[ -sin(t), cos(t)]
```

**See Also** `eig`



**Purpose** Symbolic expansion of polynomials and elementary functions

**Syntax** `expand(S)`

**Description** `expand(S)` writes each element of a symbolic expression `S` as a product of its factors. `expand` is often used with polynomials. It also expands trigonometric, exponential, and logarithmic functions.

**Examples** Expand the expression:

```
syms x;  
expand((x-2)*(x-4))
```

The result is:

```
ans =  
x^2 - 6*x + 8
```

Expand the trigonometric expression:

```
syms x y;  
expand(cos(x+y))
```

The result is:

```
ans =  
cos(x)*cos(y) - sin(x)*sin(y)
```

Expand the exponent:

```
syms a b;  
expand(exp((a+b)^2))
```

The result is:

```
ans =  
exp(2*a*b)*exp(a^2)*exp(b^2)
```

# expand

---

Expand the expressions that form a vector:

```
syms t;  
expand([sin(2*t), cos(2*t)])
```

The result is:

```
ans =  
[ 2*cos(t)*sin(t), cos(t)^2 - sin(t)^2]
```

## See Also

collect, factor, horner, simple, simplify, syms

**Purpose**

Contour plotter

**Syntax**

```
ezcontour(f)
ezcontour(f, domain)
ezcontour(..., n)
```

**Description**

`ezcontour(f)` plots the contour lines of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezcontour(f, domain)` plots  $f(x,y)$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezcontour(u^2 - v^3, [0, 1], [3, 6])` plots the contour lines for  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezcontour(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezcontour` automatically adds a title and axis labels.

**Examples**

The following mathematical expression defines a function of two variables,  $x$  and  $y$ .

$$f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) e^{-x^2-y^2} - \frac{1}{3} e^{-(x+1)^2-y^2}.$$

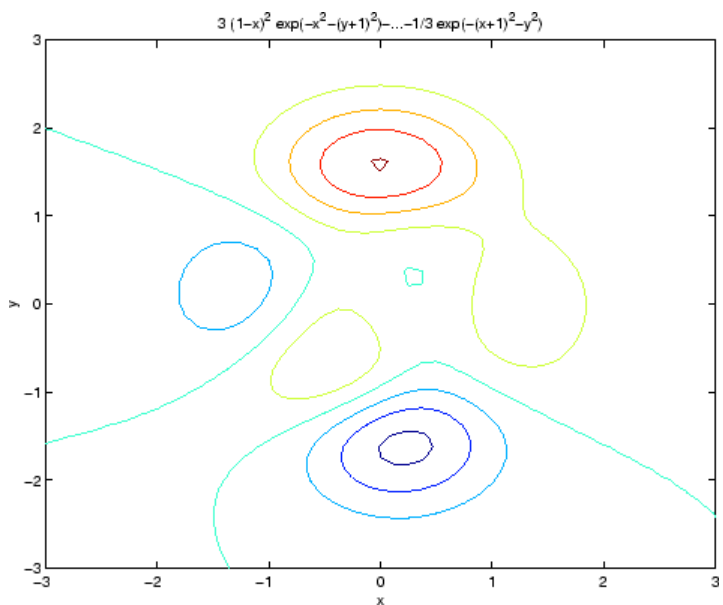
`ezcontour` requires a `sym` argument that expresses this function using MATLAB syntax to represent exponents, natural logs, etc. This function is represented by the symbolic expression

```
syms x y
f = 3*(1-x)^2*exp(-(x^2)-(y+1)^2) ...
    - 10*(x/5 - x^3 - y^5)*exp(-x^2-y^2) ...
    - 1/3*exp(-(x+1)^2 - y^2);
```

For convenience, this expression is written on three lines.

Pass the sym f to ezcontour along with a domain ranging from -3 to 3 and specify a computational grid of 49-by-49.

```
ezcontour(f, [-3,3], 49)
```



In this particular case, the title is too long to fit at the top of the graph so MATLAB abbreviates the string.

## See Also

contour, ezcontourf, ezmesh, ezmeshc, ezplot, ezplot3, ezpolar, ezsurf, ezsurf c

**Purpose**

Filled contour plotter

**Syntax**

```
ezcontour(f)
ezcontour(f, domain)
ezcontourf(..., n)
```

**Description**

`ezcontour(f)` plots the contour lines of  $f(x,y)$ , where  $f$  is a `sym` that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezcontour(f, domain)` plots  $f(x,y)$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezcontourf(u^2 - v^3, [0,1], [3,6])` plots the contour lines for  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezcontourf(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezcontourf` automatically adds a title and axis labels.

**Examples**

The following mathematical expression defines a function of two variables,  $x$  and  $y$ .

$$f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) e^{-x^2-y^2} - \frac{1}{3} e^{-(x+1)^2-y^2}.$$

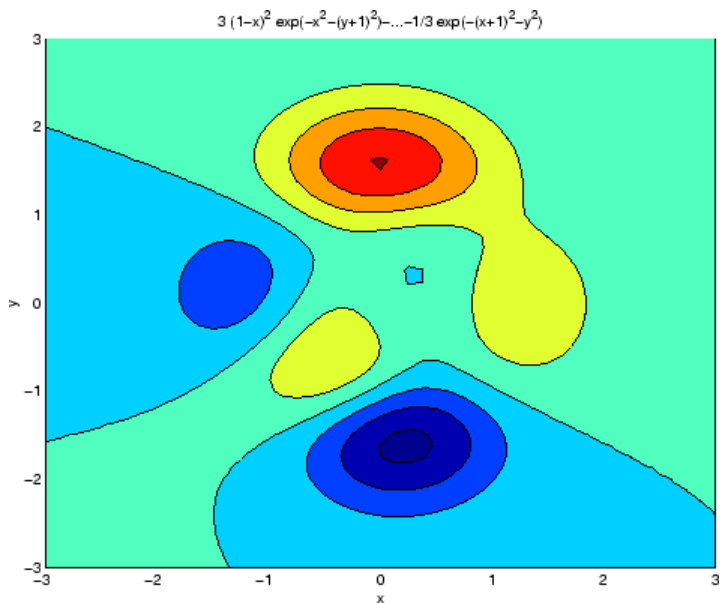
`ezcontourf` requires a `sym` argument that expresses this function using MATLAB syntax to represent exponents, natural logs, etc. This function is represented by the symbolic expression

```
syms x y
f = 3*(1-x)^2*exp(-(x^2)-(y+1)^2) ...
    - 10*(x/5 - x^3 - y^5)*exp(-x^2-y^2) ...
    - 1/3*exp(-(x+1)^2 - y^2);
```

For convenience, this expression is written on three lines.

Pass the sym f to ezcontourf along with a domain ranging from -3 to 3 and specify a grid of 49-by-49.

```
ezcontourf(f, [-3,3],49)
```



In this particular case, the title is too long to fit at the top of the graph so MATLAB abbreviates the string.

## See Also

contourf, ezcontour, ezmesh, ezmeshc, ezplot, ezplot3, ezpolar, ezsurf, ezsurf

**Purpose**

3-D mesh plotter

**Syntax**

```
ezmesh(f)
ezmesh(f, domain)
ezmesh(x,y,z)
ezmesh(x,y,z,[smin,smax,tmin,tmax])
ezmesh(x,y,z,[min,max])
ezmesh(...,n)
ezmesh(...,'circ')
```

**Description**

`ezmesh(f)` creates a graph of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezmesh(f, domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezmesh(u^2 - v^3, [0, 1], [3, 6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezmesh(x, y, z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezmesh(x, y, z, [smin, smax, tmin, tmax])` or `ezmesh(x, y, z, [min, max])` plots the parametric surface using the specified domain.

`ezmesh(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezmesh(..., 'circ')` plots  $f$  over a disk centered on the domain.

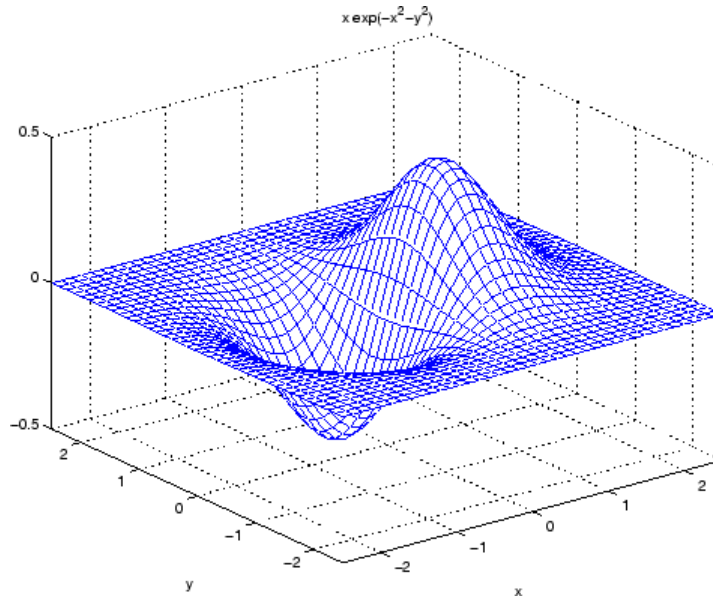
## Examples

This example visualizes the function,

$$f(x,y) = xe^{-x^2-y^2},$$

with a mesh plot drawn on a 40-by-40 grid. The mesh lines are set to a uniform blue color by setting the colormap to a single color.

```
syms x y
ezmesh(x*exp(-x^2-y^2), [-2.5,2.5],40)
colormap([0 0 1])
```



## See Also

ezcontour, ezcontourf, ezmeshc, ezplot, ezplot3, ezpolar, ezsurf, ezsurf, mesh



**Purpose**

Combined mesh and contour plotter

**Syntax**

```
ezmeshc(f)
ezmeshc(f, domain)
ezmeshc(x, y, z)
ezmeshc(x, y, z, [smin, smax, tmin, tmax])
ezmeshc(x, y, z, [min, max])
ezmeshc(..., n)
ezmeshc(..., 'circ')
```

**Description**

`ezmeshc(f)` creates a graph of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezmeshc(f, domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezmeshc(u^2 - v^3, [0, 1], [3, 6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezmeshc(x, y, z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezmeshc(x, y, z, [smin, smax, tmin, tmax])` or `ezmeshc(x, y, z, [min, max])` plots the parametric surface using the specified domain.

`ezmeshc(..., n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezmeshc(..., 'circ')` plots  $f$  over a disk centered on the domain.

## Examples

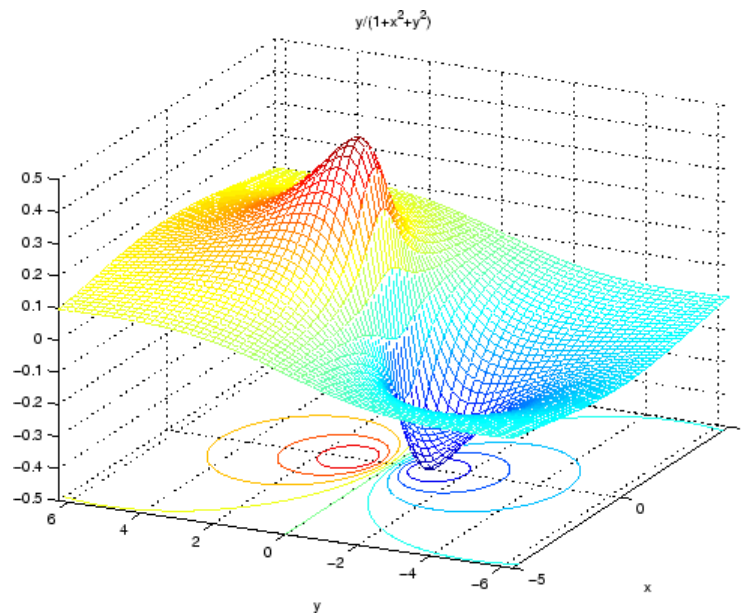
Create a mesh/contour graph of the expression,

$$f(x,y) = \frac{y}{1+x^2+y^2},$$

over the domain  $-5 < x < 5$ ,  $-2\pi < y < 2\pi$ .

```
syms x y
ezmeshc(y/(1 + x^2 + y^2), [-5,5, -2*pi,2*pi])
```

Use the mouse to rotate the axes to better observe the contour lines (this picture uses a view of azimuth = -65 and elevation = 26).



## See Also

ezcontour, ezcontourf, ezmesh, ezplot, ezplot3, ezpolar, ezsurf, ezsurf, ezsurf, meshc

**Purpose**

Function plotter

**Syntax**

```
ezplot(f)
ezplot(f,[xmin xmax])
ezplot(f,[xmin xmax], fign)
ezplot(f,[xmin, xmax, ymin, ymax])
ezplot(x,y)
ezplot(x,y,[tmin,tmax])
ezplot(...,figure)
```

**Description**

`ezplot(f)` plots the expression  $f = f(x)$  over the default domain  $-2\pi < x < 2\pi$ .

`ezplot(f,[xmin xmax])` plots  $f = f(x)$  over the specified domain. It opens and displays the result in a window labeled **Figure No. 1**. If any plot windows are already open, `ezplot` displays the result in the highest numbered window.

`ezplot(f,[xmin xmax], fign)` opens (if necessary) and displays the plot in the window labeled `fign`.

For implicitly defined functions,  $f = f(x,y)$ .

`ezplot(f)` plots  $f(x,y) = 0$  over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ .

`ezplot(f,[xmin, xmax, ymin, ymax])` plots  $f(x,y) = 0$  over  $xmin < x < xmax$  and  $ymin < y < ymax$ .

`ezplot(f,[min,max])` plots  $f(x,y) = 0$  over  $min < x < max$  and  $min < y < max$ .

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezplot(u^2 - v^2 - 1, [-3,2, -2,3])` plots  $u^2 - v^2 - 1 = 0$  over  $-3 < u < 2$ ,  $-2 < v < 3$ .

`ezplot(x,y)` plots the parametrically defined planar curve  $x = x(t)$  and  $y = y(t)$  over the default domain  $0 < t < 2\pi$ .

# ezplot

---

`ezplot(x,y,[tmin,tmax])` plots  $x = x(t)$  and  $y = y(t)$  over  $t_{\min} < t < t_{\max}$ .

`ezplot(...,figure)` plots the given function over the specified domain in the figure window identified by the handle `figure`.

## Algorithm

If you do not specify a plot range, `ezplot` samples the function between  $-2\pi$  and  $2\pi$  and selects a subinterval where the variation is significant as the plot domain. For the range, `ezplot` omits extreme values associated with singularities.

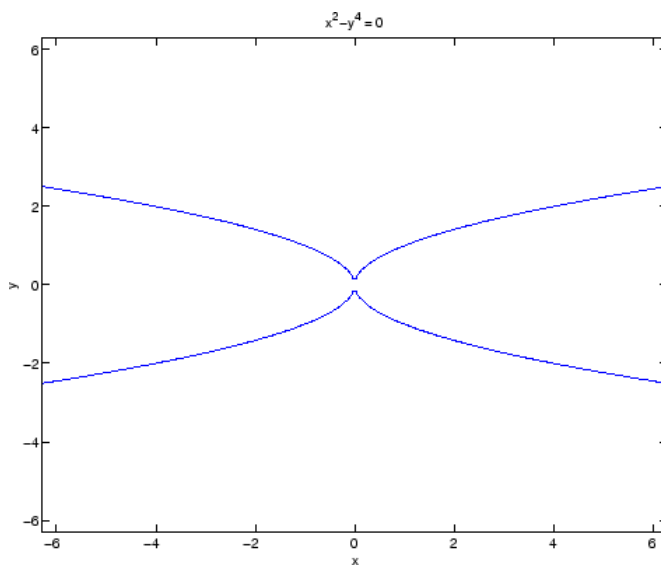
## Examples

This example plots the implicitly defined function,

$$x^2 - y^4 = 0$$

over the domain  $[-2\pi, 2\pi]$ .

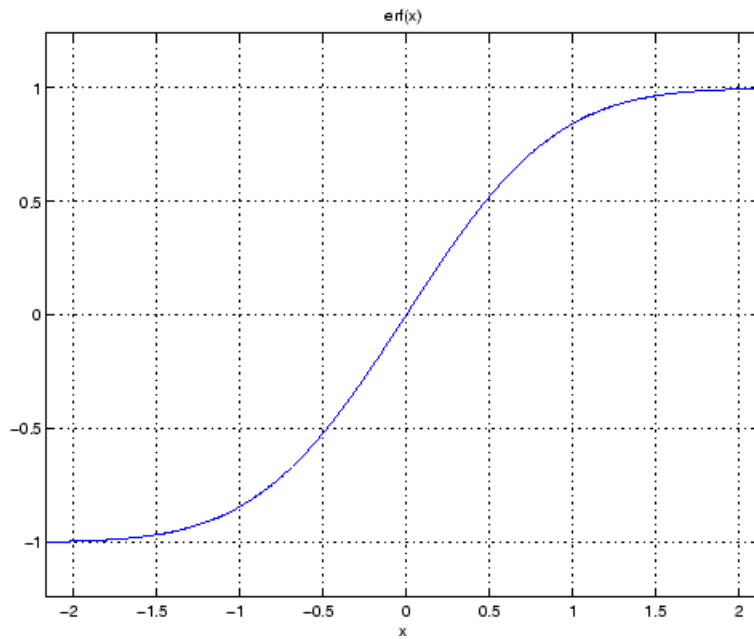
```
syms x y
ezplot(x^2-y^4)
```



The following statements

```
syms x
ezplot(erf(x))
grid
```

plot a graph of the error function.



## See Also

ezcontour, ezcontourf, ezmesh, ezmeshc, ezplot3, ezpolar, ezsurf, ezsurf, ezsurf, plot

**Purpose**

3-D parametric curve plotter

**Syntax**

```
ezplot3(x,y,z)
ezplot3(x,y,z,[tmin,tmax])
ezplot3(...,'animate')
```

**Description**

`ezplot3(x,y,z)` plots the spatial curve  $x = x(t)$ ,  $y = y(t)$ , and  $z = z(t)$  over the default domain  $0 < t < 2\pi$ .

`ezplot3(x,y,z,[tmin,tmax])` plots the curve  $x = x(t)$ ,  $y = y(t)$ , and  $z = z(t)$  over the domain  $tmin < t < tmax$ .

`ezplot3(...,'animate')` produces an animated trace of the spatial curve.

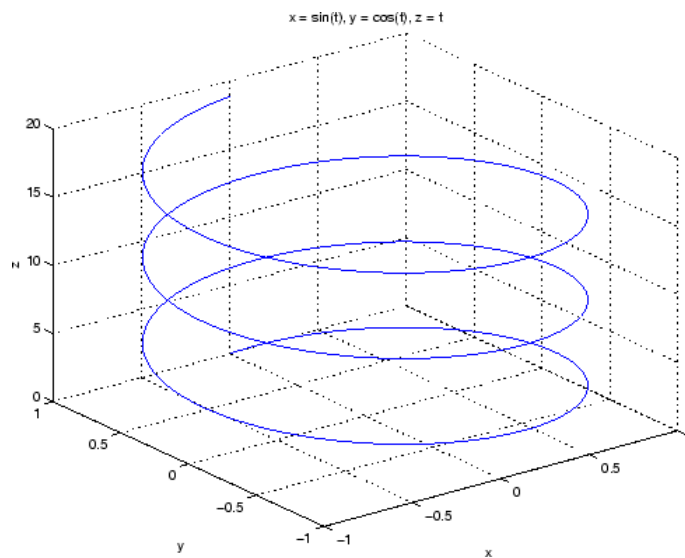
**Examples**

This example plots the parametric curve,  $x = \sin(t)$ ,  $y = \cos(t)$ ,  $z = t$  over the domain  $[0, 6\pi]$ .

```
syms t;
ezplot3(sin(t), cos(t), t,[0,6*pi])
```

# ezplot3

---



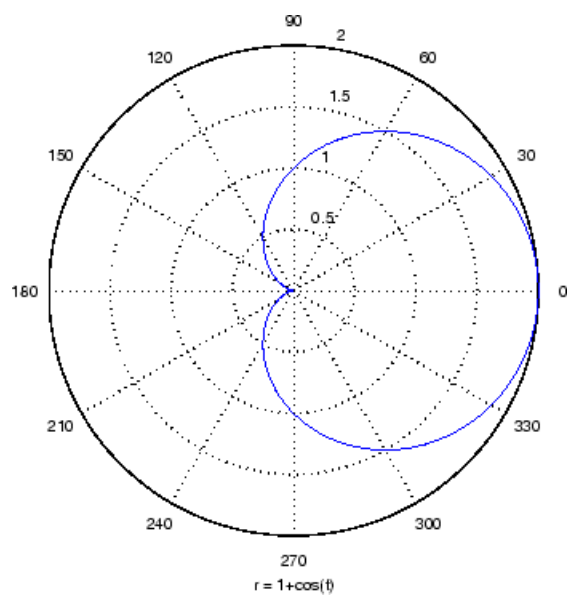
## See Also

ezcontour, ezcontourf, ezmesh, ezmeshc, ezplot, ezpolar, ezsurf, ezsurf, ezsurf, plot3



---

<b>Purpose</b>	Polar coordinate plotter
<b>Syntax</b>	<code>ezpolar(f)</code> <code>ezpolar(f, [a, b])</code>
<b>Description</b>	<code>ezpolar(f)</code> plots the polar curve $r = f(\theta)$ over the default domain $0 < \theta < 2\pi$ . <code>ezpolar(f, [a, b])</code> plots $f$ for $a < \theta < b$ .
<b>Example</b>	This example creates a polar plot of the function, $1 + \cos(t)$ over the domain $[0, 2\pi]$ .  <pre>syms t ezpolar(1 + cos(t))</pre>



**Purpose**

3-D colored surface plotter

**Syntax**

```
ezsurf(f)
ezsurf(f, domain)
ezsurf(x,y,z)
ezsurf(x,y,z,[smin,smax,tmin,tmax])
ezsurf(x,y,z,[min,max])
ezsurf(...,n)
ezsurf(...,'circ')
```

`ezsurf(f)` plots over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezsurf(f, domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezsurf(u^2 - v^3, [0, 1], [3, 6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezsurf(x,y,z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezsurf(x,y,z,[smin,smax,tmin,tmax])` or `ezsurf(x,y,z,[min,max])` plots the parametric surface using the specified domain.

`ezsurf(...,n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezsurf(...,'circ')` plots  $f$  over a disk centered on the domain.

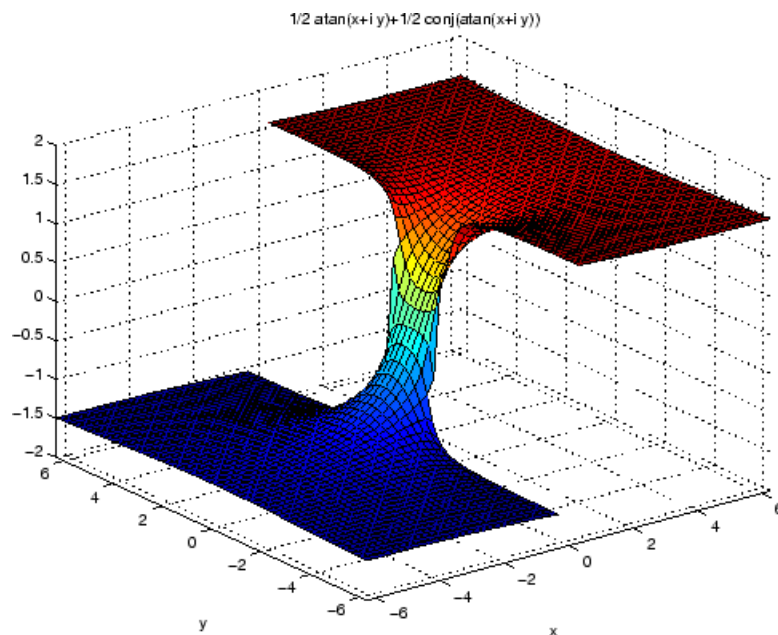
**Examples**

`ezsurf` does not graph points where the mathematical function is not defined (these data points are set to NaNs, which MATLAB does not plot). This example illustrates this filtering of singularities/discontinuous points by graphing the function,

$$f(x,y) = \text{real}(\text{atan}(x + iy))$$

over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ .

```
syms x y
ezsurf(real(atan(x+i*y)))
```



Note also that `ezsurf` creates graphs that have axis labels, a title, and extend to the axis limits.

## See Also

`ezcontour`, `ezcontourf`, `ezmesh`, `ezmeshc`, `ezplot`, `ezpolar`, `ezsurf`, `ezsurf`, `surf`

**Purpose**

Combined surface and contour plotter

**Syntax**

```
ezsurf(f)
ezsurf(f,domain)
ezsurf(x,y,z)
ezsurf(x,y,z,[smin,smax,tmin,tmax])
ezsurf(x,y,z,[min,max])
ezsurf(...,n)
ezsurf(...,'circ')
```

**Description**

`ezsurf(f)` creates a graph of  $f(x,y)$ , where  $f$  is a symbolic expression that represents a mathematical function of two variables, such as  $x$  and  $y$ .

The function  $f$  is plotted over the default domain  $-2\pi < x < 2\pi$ ,  $-2\pi < y < 2\pi$ . MATLAB software chooses the computational grid according to the amount of variation that occurs; if the function  $f$  is not defined (singular) for points on the grid, then these points are not plotted.

`ezsurf(f,domain)` plots  $f$  over the specified domain. `domain` can be either a 4-by-1 vector  $[xmin, xmax, ymin, ymax]$  or a 2-by-1 vector  $[min, max]$  (where,  $min < x < max$ ,  $min < y < max$ ).

If  $f$  is a function of the variables  $u$  and  $v$  (rather than  $x$  and  $y$ ), then the domain endpoints  $umin$ ,  $umax$ ,  $vmin$ , and  $vmax$  are sorted alphabetically. Thus, `ezsurf(u^2 - v^3, [0, 1], [3, 6])` plots  $u^2 - v^3$  over  $0 < u < 1$ ,  $3 < v < 6$ .

`ezsurf(x,y,z)` plots the parametric surface  $x = x(s,t)$ ,  $y = y(s,t)$ , and  $z = z(s,t)$  over the square  $-2\pi < s < 2\pi$ ,  $-2\pi < t < 2\pi$ .

`ezsurf(x,y,z,[smin,smax,tmin,tmax])` or `ezsurf(x,y,z,[min,max])` plots the parametric surface using the specified domain.

`ezsurf(...,n)` plots  $f$  over the default domain using an  $n$ -by- $n$  grid. The default value for  $n$  is 60.

`ezsurf(...,'circ')` plots  $f$  over a disk centered on the domain.

## Examples

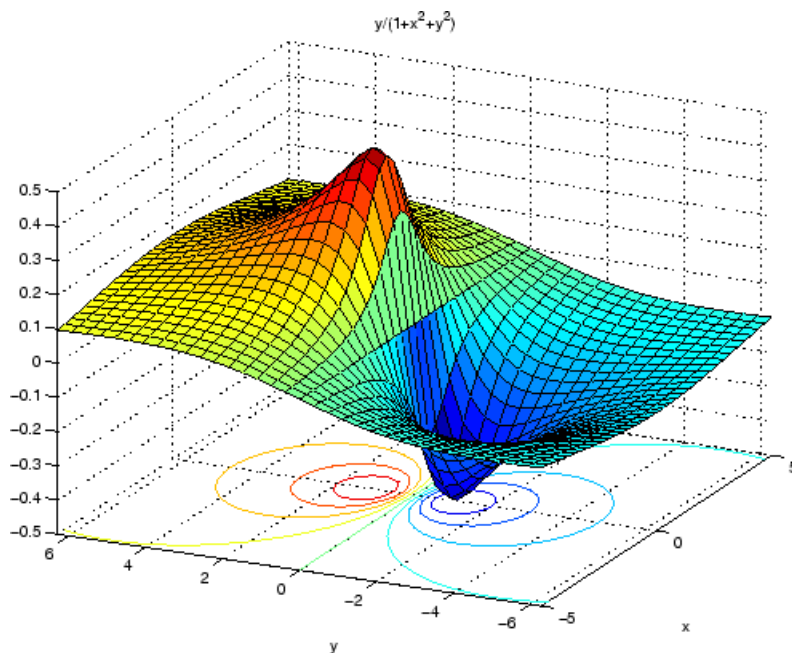
Create a surface/contour plot of the expression,

$$f(x, y) = \frac{y}{1 + x^2 + y^2},$$

over the domain  $-5 < x < 5$ ,  $-2\pi < y < 2\pi$ , with a computational grid of size 35-by-35

```
syms x y
ezsurf(y/(1 + x^2 + y^2), [-5,5, -2*pi,2*pi], 35)
```

Use the mouse to rotate the axes to better observe the contour lines (this picture uses a view of azimuth = -65 and elevation = 26).



**See Also**

ezcontour, ezcontourf, ezmesh, ezmeshc, ezplot, ezpolar, ezsurf,  
surf

# factor

---

**Purpose** Factorization

**Syntax** `factor(X)`

**Description** `factor(X)` can take a positive integer, an array of symbolic expressions, or an array of symbolic integers as an argument. If  $N$  is a positive integer, `factor(N)` returns the prime factorization of  $N$ .

If  $S$  is a matrix of polynomials or integers, `factor(S)` factors each element. If any element of an integer array has more than 16 digits, you must use `sym` to create that element, for example, `sym('N')`.

**Examples** Factorize the two-variable expression:

```
syms x y;  
factor(x^3-y^3)
```

The result is:

```
ans =  
(x - y)*(x^2 + x*y + y^2)
```

Factorize the expressions that form a vector:

```
syms a b;  
factor([a^2 - b^2, a^3 + b^3])
```

The result is:

```
ans =  
[ (a - b)*(a + b), (a + b)*(a^2 - a*b + b^2)]
```

Factorize the symbolic number:

```
factor(sym('12345678901234567890'))
```

The result is:



```
ans =  
2*3^2*5*101*3541*3607*3803*27961
```

**See Also**

collect, expand, horner, simplify, simple

# feval

---

**Purpose** Evaluate MuPAD expressions

**Syntax**  
`result = feval(symengine,F,x1,...,xn)`  
`[result,status] = feval(symengine,F,x1,...,xn)`

**Description** `result = feval(symengine,F,x1,...,xn)` evaluates `F`, which is either a MuPAD function name or a symbolic object, with arguments `x1,...,xn`, with `result` a symbolic object.

`[result,status] = feval(symengine,F,x1,...,xn)` returns the error status in `status`, and the error message in `g` if `status` is nonzero. If `status` is 0, `result` is a symbolic object; otherwise, it `g` is a string.

## Examples

```
syms a b c x
p = a*x^2+b*x+c;
feval(symengine,'polylib::discrim', p, x)
```

```
ans =
b^2 - 4*a*c
```

Alternatively, the same calculation based on variables not defined in the MATLAB workspace is:

```
feval(symengine,'polylib::discrim', 'a*x^2
+ b*x + c', 'x')
```

```
ans =
b^2 - 4*a*c
```

## See Also

`doc, evalin`

“Calling MuPAD Functions at the MATLAB Command Line” on page 4-27

**Purpose**

Determine variables in symbolic expression or matrix

**Syntax**

```
findsym(S)  
findsym(S,n)
```

**Description**

`findsym(S)` for a symbolic expression or matrix `S`, returns all symbolic variables in `S` in lexicographical order, separated by commas. If `S` does not contain any variables, `findsym` returns an empty string.

`findsym(S,n)` returns the `n` variables alphabetically closest to `x`:

- 1** The variables are sorted by the first letters in their names. The ordering is `x y w z v u ... a X Y W Z V U ... A`. The name of a symbolic variable cannot begin with a number.
- 2** For all subsequent letters the ordering is alphabetical, with all uppercase letters having precedence over lowercase: `0 1 ... 9 A B ... Z a b ...z`.

---

**Note** `findsym(S)` can return variables in different order than `findsym(S,n)`.

---

**Examples**

```
syms a x y z t X1 x2 xa xb  
findsym(sin(pi*t))
```

```
ans =  
t
```

```
findsym(x+i*y-j*z)
```

```
ans =  
x,y,z
```

```
findsym(a+y,1)
```

# findsym

---

```
ans =  
y
```

```
findsym(X1 + x2 + xa + xb)
```

```
ans =  
X1,x2,xa,xb
```

```
findsym(X1 + x2 + xa + xb,4)
```

```
ans =  
x2,xa,xb,X1
```

## See Also

compose, diff, int, limit, symvar, taylor

**Purpose** Functional inverse

**Syntax**  
`g = finverse(f)`  
`g = finverse(f,v)`

**Description** `g = finverse(f)` returns the functional inverse of `f`. `f` is a scalar sym representing a function of one symbolic variable, say `x`. Then `g` is a scalar sym that satisfies  $g(f(x)) = x$ . That is, `finverse(f)` returns  $f^{-1}$ , provided  $f^{-1}$  exists.

`g = finverse(f,v)` uses the symbolic variable `v`, where `v` is a sym, as the independent variable. Then `g` is a scalar sym that satisfies  $g(f(v)) = v$ . Use this form when `f` contains more than one symbolic variable.

**Examples** Compute functional inverse for the trigonometric function:

```
syms x u v;
finverse(1/tan(x))
```

The result is:

```
ans =
atan(1/x)
```

Compute functional inverse for the exponent function:

```
finverse(exp(u - 2*v), u)
```

The result is:

```
ans =
2*v + log(u)
```

**See Also** `compose`, `syms`

# fix

---

**Purpose**            Round toward zero

**Syntax**            `fix(X)`

**Description**        `fix(X)` is the matrix of the integer parts of  $X$ .  
`fix(X) = floor(X)` if  $X$  is positive and `ceil(X)` if  $X$  is negative.

**See Also**            `round`, `ceil`, `floor`, `frac`

**Purpose** Round symbolic matrix toward negative infinity

**Syntax** `floor(X)`

**Description** `floor(X)` is the matrix of the greatest integers less than or equal to  $X$ .

**Example**

```
x = sym(-5/2);  
[fix(x) floor(x) round(x) ceil(x) frac(x)]  
  
ans =  
[ -2, -3, -3, -2, -1/2]
```

**See Also** `round`, `ceil`, `fix`, `frac`

# fortran

---

**Purpose** Fortran representation of symbolic expression

**Syntax** `fortran(S)`  
`fortran(S, 'file', fileName)`

**Description** `fortran(S)` returns the Fortran code equivalent to the expression `S`.  
`fortran(S, 'file', fileName)` writes an “optimized” Fortran code fragment that evaluates the symbolic expression `S` to the file named `fileName`. “Optimized” means intermediate variables are automatically generated in order to simplify the code.

**Examples** The statements

```
syms x
f = taylor(log(1+x));
fortran(f)

return

ans =
    t0 = x-x**2*(1.000/2.000)+x**3*(1.000/3.000)-x**4*(1.000/4.000)+x*
    ~*5*(1.000/5.000)
```

The statements

```
H = sym(hilb(3));
fortran(H)

return

ans =
    H(1,1) = 1
    H(1,2) = 1.000/2.000
    H(1,3) = 1.000/3.000
    H(2,1) = 1.000/2.000
    H(2,2) = 1.000/3.000
```



```
H(2,3) = 1.0D0/4.0D0
H(3,1) = 1.0D0/3.0D0
H(3,2) = 1.0D0/4.0D0
H(3,3) = 1.0D0/5.0D0
```

The statements

```
syms x
z = exp(-exp(-x));
fortran(diff(z,3), 'file', 'fortrantest');
```

return a file named `fortrantest` containing the following:

```
t7 = exp(-x)
t8 = exp(-t7)
t0 = t8*exp(x*(-2))*(-3)+t8*exp(x*(-3))+t7*t8
```

## See Also

`ccode`, `latex`, `matlabFunction`, `pretty`

“Generating Code from Symbolic Expressions” on page 3-129

# fourier

---

**Purpose**            Fourier integral transform

**Syntax**             $F = \text{fourier}(f)$   
                       $F = \text{fourier}(f, v)$   
                       $F = \text{fourier}(f, u, v)$

**Description**         $F = \text{fourier}(f)$  is the Fourier transform of the symbolic scalar  $f$  with default independent variable  $x$ . The default return is a function of  $w$ . The Fourier transform is applied to a function of  $x$  and returns a function of  $w$ .

$$f = f(x) \Rightarrow F = F(w)$$

If  $f = f(w)$ , `fourier` returns a function of  $t$ .

$$F = F(t)$$

By definition,

$$F(w) = \int_{-\infty}^{\infty} f(x)e^{-iwx} dx$$

where  $x$  is the symbolic variable in  $f$  as determined by `symvar`.

$F = \text{fourier}(f, v)$  makes  $F$  a function of the symbol  $v$  instead of the default  $w$ .

$$F(v) = \int_{-\infty}^{\infty} f(x)e^{-ivx} dx$$

$F = \text{fourier}(f, u, v)$  makes  $f$  a function of  $u$  and  $F$  a function of  $v$  instead of the default variables  $x$  and  $w$ , respectively.

$$F(v) = \int_{-\infty}^{\infty} f(u)e^{-ivu} du$$

Examples

Fourier Transform	MATLAB Commands
$f(x) = e^{-x^2}$ $F[f](w) = \int_{-\infty}^{\infty} f(x)e^{-ixw} dx$ $= \sqrt{\pi} e^{-w^2/4}$	<pre>syms x; f = exp(-x^2); fourier(f)</pre> <p>returns</p> <pre>ans = pi^(1/2)/exp(w^2/4)</pre>
$g(w) = e^{- w }$ $F[g](t) = \int_{-\infty}^{\infty} g(w)e^{-itw} dw$ $= \frac{2}{1+t^2}$	<pre>syms w; g = exp(-abs(w)); fourier(g)</pre> <p>returns</p> <pre>ans = 2/(v^2 + 1)</pre>
$f(x) = xe^{- x }$ $F[f](u) = \int_{-\infty}^{\infty} f(x)e^{-ixu} dx$ $= -\frac{4iu}{(1+u^2)^2}$	<pre>syms x u; f = x*exp(-abs(x)); fourier(f,u)</pre> <p>returns</p> <pre>ans = -(4*i*u)/(u^2 + 1)^2</pre>
$f(x,v) = e^{-x^2 \frac{ v  \sin v}{v}}, x \text{ real}$	<pre>syms v u; syms x real;</pre>

# fourier

---

Fourier Transform	MATLAB Commands
$F[f(v)](u) = \int_{-\infty}^{\infty} f(x,v)e^{-ivu} dv$ $= -\arctan \frac{u-1}{x^2} + \arctan \frac{u+1}{x^2}$	<pre>f = exp(-x^2*abs(v))*sin(v)/v; fourier(f,v,u)  returns  ans = piecewise([x &lt;&gt; 0,... atan((u + 1)/x^2)... - atan(1/x^2*(u - 1))])</pre>

## See Also

ifourier, laplace, ztrans

---

<b>Purpose</b>	Symbolic matrix elementwise fractional parts
<b>Syntax</b>	<code>frac(X)</code>
<b>Description</b>	<code>frac(X)</code> is the matrix of the fractional parts of the elements: <code>frac(X) = X - fix(X)</code> .
<b>Example</b>	<pre>x = sym(-5/2); [fix(x) floor(x) round(x) ceil(x) frac(x)]  ans = [ -2, -3, -3, -2, -1/2]</pre>
<b>See Also</b>	<code>round</code> , <code>ceil</code> , <code>floor</code> , <code>fix</code>

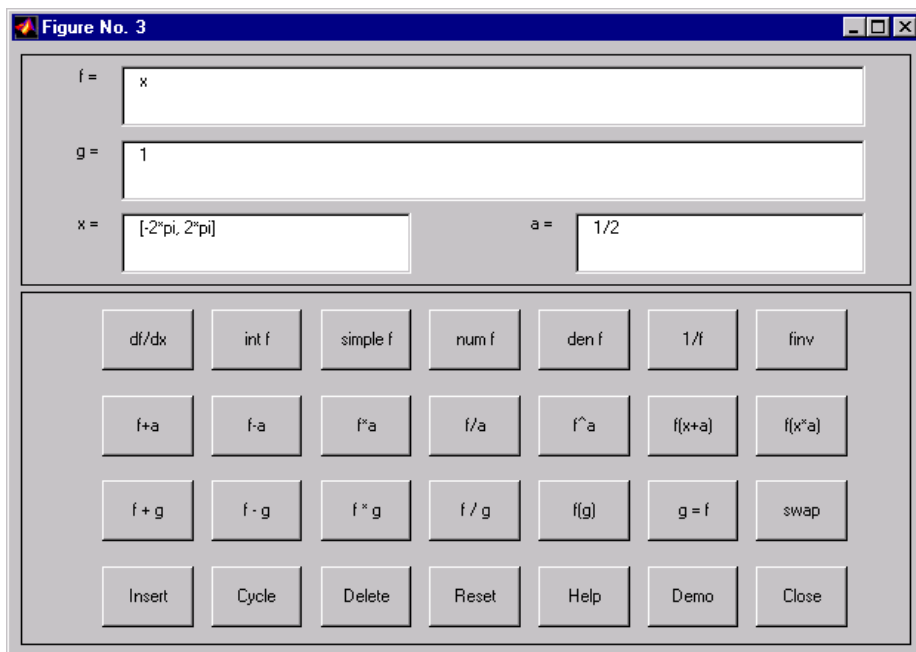
# funtool

**Purpose** Function calculator

**Syntax** funtool

**Description** funtool is a visual function calculator that manipulates and displays functions of one variable. At the click of a button, for example, funtool draws a graph representing the sum, product, difference, or ratio of two functions that you specify. funtool includes a function memory that allows you to store functions for later retrieval.

At startup, funtool displays graphs of a pair of functions,  $f(x) = x$  and  $g(x) = 1$ . The graphs plot the functions over the domain  $[-2\pi, 2\pi]$ . funtool also displays a control panel that lets you save, retrieve, redefine, combine, and transform  $f$  and  $g$ .



## Text Fields

The top of the control panel contains a group of editable text fields.

<b>f=</b>	Displays a symbolic expression representing $f$ . Edit this field to redefine $f$ .
<b>g=</b>	Displays a symbolic expression representing $g$ . Edit this field to redefine $g$ .
<b>x=</b>	Displays the domain used to plot $f$ and $g$ . Edit this field to specify a different domain.
<b>a=</b>	Displays a constant factor used to modify $f$ (see button descriptions in the next section). Edit this field to change the value of the constant factor.

funtool redraws  $f$  and  $g$  to reflect any changes you make to the contents of the control panel's text fields.

## Control Buttons

The bottom part of the control panel contains an array of buttons that transform  $f$  and perform other operations.

The first row of control buttons replaces  $f$  with various transformations of  $f$ .

<b>df/dx</b>	Derivative of $f$
<b>int f</b>	Integral of $f$
<b>simple f</b>	Simplified form of $f$ , if possible
<b>num f</b>	Numerator of $f$
<b>den f</b>	Denominator of $f$
<b>1/f</b>	Reciprocal of $f$
<b>finv</b>	Inverse of $f$

The operators **intf** and **finv** may fail if the corresponding symbolic expressions do not exist in closed form.

The second row of buttons translates and scales  $f$  and the domain of  $f$  by a constant factor. To specify the factor, enter its value in the field labeled **a=** on the calculator control panel. The operations are

<b>f+a</b>	Replaces $f(x)$ by $f(x) + a$ .
<b>f-a</b>	Replaces $f(x)$ by $f(x) - a$ .
<b>f*a</b>	Replaces $f(x)$ by $f(x) * a$ .
<b>f/a</b>	Replaces $f(x)$ by $f(x) / a$ .
<b>f^a</b>	Replaces $f(x)$ by $f(x) ^ a$ .
<b>f(x+a)</b>	Replaces $f(x)$ by $f(x + a)$ .
<b>f(x*a)</b>	Replaces $f(x)$ by $f(x * a)$ .

The first four buttons of the third row replace  $f$  with a combination of  $f$  and  $g$ .

<b>f+g</b>	Replaces $f(x)$ by $f(x) + g(x)$ .
<b>f-g</b>	Replaces $f(x)$ by $f(x) - g(x)$ .
<b>f*g</b>	Replaces $f(x)$ by $f(x) * g(x)$ .
<b>f/g</b>	Replaces $f(x)$ by $f(x) / g(x)$ .

The remaining buttons on the third row interchange  $f$  and  $g$ .

<b>g=f</b>	Replaces $g$ with $f$ .
<b>swap</b>	Replaces $f$ with $g$ and $g$ with $f$ .

The first three buttons in the fourth row allow you to store and retrieve functions from the calculator's function memory.

<b>Insert</b>	Adds $f$ to the end of the list of stored functions.
<b>Cycle</b>	Replaces $f$ with the next item on the function list.
<b>Delete</b>	Deletes $f$ from the list of stored functions.



The other four buttons on the fourth row perform miscellaneous functions:

<b>Reset</b>	Resets the calculator to its initial state.
<b>Help</b>	Displays the online help for the calculator.
<b>Demo</b>	Runs a short demo of the calculator.
<b>Close</b>	Closes the calculator's windows.

**See Also**      ezplot, syms

# getVar

---

**Purpose** Get variable from MuPAD notebook

**Syntax** `y = getVar(nb, 'z')`

**Description** `y = getVar(nb, 'z')` assigns the symbolic expression `z` in the MuPAD notebook `nb` to a symbolic variable `y` in the MATLAB workspace.

## Example

```
mpnb = mupad;  
% make a variable f:=x^2 in the MuPAD notebook  
f = getVar(mpnb, 'f')
```

`f` is a symbolic variable in the MATLAB workspace, with value  $x^2$ .

**See Also** `mupad`, `setVar`

<b>Purpose</b>	Compute Heaviside step function
<b>Syntax</b>	<code>heaviside(x)</code>
<b>Description</b>	<code>heaviside(x)</code> has the value 0 for $x < 0$ , 1 for $x > 0$ , and 0.5 for $x = 0$ .
<b>Examples</b>	<p>For <math>x &lt; 0</math> the function <code>heaviside(x)</code> returns 0:</p> <pre>heaviside(sym(-3))  ans = 0</pre> <p>For <math>x &gt; 0</math> the function, <code>heaviside(x)</code> returns 1:</p> <pre>heaviside(sym(3))  ans = 1</pre> <p>For <math>x = 0</math> the function, <code>heaviside(x)</code> returns 1/2:</p> <pre>heaviside(sym(0))  ans = 1/2</pre> <p>For numeric <math>x = 0</math> the function, <code>heaviside(x)</code> returns the numeric result:</p> <pre>heaviside(0)  ans = 0.5000</pre>
<b>See Also</b>	<code>dirac</code>

# horner

---

**Purpose** Horner nested polynomial representation

**Syntax** horner(P)

**Description** Suppose P is a matrix of symbolic polynomials. horner(P) transforms each element of P into its Horner, or nested, representation.

**Examples** Find nested polynomial representation of the polynomial:

```
syms x
horner(x^3-6*x^2+11*x-6)
```

The result is

```
ans =
x*(x*(x - 6) + 11) - 6
```

Find nested polynomial representation for the polynomials that form a vector:

```
syms x y
horner([x^2+x;y^3-2*y])
```

The result is:

```
ans =
x*(x + 1)
y*(y^2 - 2)
```

**See Also** expand, factor, simple, simplify, syms

**Purpose** Generalized hypergeometric

**Syntax** hypergeom(n, d, z)

**Description** hypergeom(n, d, z) is the generalized hypergeometric function  $F(n, d, z)$ , also known as the Barnes extended hypergeometric function and denoted by  ${}_jF_k$  where  $j = \text{length}(n)$  and  $k = \text{length}(d)$ . For scalar a, b, and c, hypergeom([a, b], c, z) is the Gauss hypergeometric function  ${}_2F_1(a, b; c; z)$ .

The definition by a formal power series is

$$F(n, d, z) = \sum_{k=0}^{\infty} \frac{C_{n,k}}{C_{d,k}} \cdot \frac{z^k}{k!},$$

where

$$C_{v,k} = \prod_{j=1}^{|v|} \frac{\Gamma(v_j + k)}{\Gamma(v_j)}.$$

Either of the first two arguments may be a vector providing the coefficient parameters for a single function evaluation. If the third argument is a vector, the function is evaluated pointwise. The result is numeric if all the arguments are numeric and symbolic if any of the arguments is symbolic.

See Abramowitz and Stegun, *Handbook of Mathematical Functions*, Chapter 15.

**Examples** Compute hypergeometric functions:

```
syms a z
q = hypergeom([], [], z)
r = hypergeom(1, [], z)
s = hypergeom(a, [], z)
```

# hypergeom

---

The results are:

$$q = \exp(z)$$

$$r = -1/(z - 1)$$

$$s = 1/(1 - z)^a$$

**Purpose** Inverse Fourier integral transform

**Syntax**

```
f = ifourier(F)
f = ifourier(F,u)
f = ifourier(F,v,u)
```

**Description** `f = ifourier(F)` is the inverse Fourier transform of the scalar symbolic object `F` with default independent variable `w`. The default return is a function of `x`. The inverse Fourier transform is applied to a function of `w` and returns a function of `x`.

$$F = F(w) \Rightarrow f = f(x).$$

If `F = F(x)`, `ifourier` returns a function of `t`:

$$f = f(t)$$

By definition

$$f(x) = 1/(2\pi) \int_{-\infty}^{\infty} F(w)e^{iwx} dw.$$

`f = ifourier(F,u)` makes `f` a function of `u` instead of the default `x`.

$$f(u) = 1/(2\pi) \int_{-\infty}^{\infty} F(w)e^{iwu} dw.$$

Here `u` is a scalar symbolic object.

`f = ifourier(F,v,u)` takes `F` to be a function of `v` and `f` to be a function of `u` instead of the default `w` and `x`, respectively.

$$f(u) = 1/(2\pi) \int_{-\infty}^{\infty} F(v)e^{ivu} dv.$$

## Examples

Inverse Fourier Transform	MATLAB Commands
$f(w) = e^{-w^2/(4a^2)}$ $F^{-1}[f](x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{ixw} dw$ $= \frac{ a }{\sqrt{\pi}} e^{-(ax)^2}$	<pre>syms a w real; f = exp(-w^2/(4*a^2)); F = ifourier(f); F = simple(F)</pre> <p>returns</p> <p>F = abs(a)/(pi^(1/2)*exp(a^2*x^2))</p>
$g(x) = e^{- x }$ $F^{-1}[g](t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(x)e^{itx} dx$ $= \frac{1}{\pi(1+t^2)}$	<pre>syms x real; g = exp(-abs(x)); ifourier(g)</pre> <p>returns</p> <p>ans = 1/(pi*(t^2 + 1))</p>
$f(w) = 2e^{- w } - 1$ $F^{-1}[f](t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(w)e^{itw} dw$ $= -\text{dirac}(t) + \frac{2}{\pi(1+t^2)}$	<pre>syms w t real; f = 2*exp(-abs(w)) - 1; simplify(ifourier(f,t))</pre> <p>returns</p> <p>ans = 2/(pi*(t^2 + 1)) - dirac(t)</p>

## See Also

fourier, ilaplace, iztrans



**Purpose** Inverse Laplace transform

**Syntax**

```
F = ilaplace(L)
F = ilaplace(L,y)
F = ilaplace(L,y,x)
```

**Description**  $F = \text{ilaplace}(L)$  is the inverse Laplace transform of the scalar symbolic object  $L$  with default independent variable  $s$ . The default return is a function of  $t$ . The inverse Laplace transform is applied to a function of  $s$  and returns a function of  $t$ .

$$L = L(s) \Rightarrow F = F(t)$$

If  $L = L(t)$ , `ilaplace` returns a function of  $x$ .

$$F = F(x)$$

By definition

$$F(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} L(s)e^{st} ds,$$

where  $c$  is a real number selected so that all singularities of  $L(s)$  are to the left of the line  $s = c$ ,  $i$ .

$F = \text{ilaplace}(L,y)$  makes  $F$  a function of  $y$  instead of the default  $t$ .

$$F(y) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} L(y)e^{sy} ds$$

Here  $y$  is a scalar symbolic object.

$F = \text{ilaplace}(L,y,x)$  takes  $F$  to be a function of  $x$  and  $L$  a function of  $y$  instead of the default variables  $t$  and  $s$ , respectively.

# ilaplace

$$F(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} L(y)e^{xy} dy$$

## Examples

Inverse Laplace Transform	MATLAB Command
$f(s) = \frac{1}{s^2}$ $L^{-1}[f] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} f(s)e^{st} ds$ $= t$	<pre>syms s; f = 1/s^2; ilaplace(f)</pre> <p>returns</p> <p>ans = t</p>
$g(t) = \frac{1}{(t-a)^2}$ $L^{-1}[g] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(t)e^{xt} dt$ $= xe^{ax}$	<pre>syms a t; g = 1/(t-a)^2; ilaplace(g)</pre> <p>returns</p> <p>ans = x*exp(a*x)</p>
$f(u) = \frac{1}{u^2 - a^2}$ $L^{-1}[f] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(u)e^{xu} du$ $= \frac{\sinh(xa)}{a}$	<pre>syms x u; syms a real; f = 1/(u^2-a^2); simplify(ilaplace(f,x))</pre> <p>returns</p> <p>ans = sinh(a*x)/a</p>

**See Also**      ifourier, iztrans, laplace

# imag

---

**Purpose**            Imaginary part of complex number

**Syntax**            `imag(Z)`

**Description**        `imag(Z)` is the imaginary part of a symbolic  $Z$ .

**See Also**            `conj`, `real`

**Purpose** Integrate symbolic expression

**Syntax**

```
int(expr)
int(expr, v)
int(expr, a, b)
int(expr, v, a, b)
```

**Description**

`int(expr)` returns the indefinite integral of `expr` with respect to its symbolic variable as defined by `symvar`.

`int(expr, v)` returns the indefinite integral of `expr` with respect to the symbolic scalar variable `v`.

`int(expr, a, b)` returns the definite integral from `a` to `b` of `expr` with respect to the default symbolic variable. `a` and `b` are symbolic or double scalars.

`int(expr, v, a, b)` returns the definite integral of `expr` with respect to `v` from `a` to `b`.

**Examples** Find indefinite integral of the following single-variable expression:

```
syms x;
int(-2*x/(1 + x^2)^2)
```

The result is:

```
ans =
1/(x^2 + 1)
```

---

Find indefinite integral of the following multivariable expression with respect to `z`:

```
syms x z;
int(x/(1 + z^2), z)
```

The result is:

```
ans =  
x*atan(z)
```

---

Integral the following expression from 0 to 1:

```
syms x;  
int(x*log(1 + x), 0, 1)
```

The result is:

```
ans =  
1/4
```

---

Integral the following expression from  $\sin(t)$  to 1:

```
syms x t;  
int(2*x, sin(t), 1)
```

The result is:

```
ans =  
cos(t)^2
```

---

Find indefinite integrals for the expressions listed as the elements of a matrix:

```
syms x t z;  
alpha = sym('alpha');  
int([exp(t), exp(alpha*t)])
```

The result is:

```
ans =  
[ exp(t), exp(alpha*t)/alpha]
```

**See Also**      `diff` | `symsum` | `symvar`

# int8, int16, int32, int64

---

**Purpose** Convert symbolic matrix to signed integers

**Syntax** int8(S)  
int16(S)  
int32(S)  
int64(S)

**Description** int8(S) converts a symbolic matrix S to a matrix of signed 8-bit integers.  
int16(S) converts S to a matrix of signed 16-bit integers.  
int32(S) converts S to a matrix of signed 32-bit integers.  
int64(S) converts S to a matrix of signed 64-bit integers.

---

**Note** The output of int8, int16, int32, and int64 does not have data type symbolic.

---

The following table summarizes the output of these four functions.

Function	Output Range	Output Type	Bytes per Element	Output Class
int8	-128 to 127	Signed 8-bit integer	1	int8
int16	-32,768 to 32,767	Signed 16-bit integer	2	int16
int32	-2,147,483,648 to 2,147,483,647	Signed 32-bit integer	4	int32
int64	-9,223,372,036,854,775,808 to 9,223,372,036,854,775,807	Signed 64-bit integer	8	int64

**See Also** sym, vpa, single, double, uint8, uint16, uint32, uint64



---

<b>Purpose</b>	Compute symbolic matrix inverse
<b>Syntax</b>	<code>R = inv(A)</code>
<b>Description</b>	<code>R = inv(A)</code> returns inverse of the symbolic matrix A.
<b>Examples</b>	Compute the inverse of the following matrix of symbolic numbers:

```
A = sym([2, -1, 0; -1, 2, -1; 0, -1, 2]);  
inv(A)
```

The result is:

```
ans =  
[ 3/4, 1/2, 1/4]  
[ 1/2, 1, 1/2]  
[ 1/4, 1/2, 3/4]
```

---

Compute the inverse of the following symbolic matrix:

```
syms a b c d  
A = [a b; c d];  
inv(A)
```

The result is:

```
ans =  
[ d/(a*d - b*c), -b/(a*d - b*c)]  
[ -c/(a*d - b*c), a/(a*d - b*c)]
```

---

Compute the inverse of the symbolic Hilbert matrix:

```
inv(sym(hilb(4)))
```

The result is:

```
ans =  
[ 16, -120, 240, -140]  
[ -120, 1200, -2700, 1680]  
[ 240, -2700, 6480, -4200]  
[ -140, 1680, -4200, 2800]
```

## See Also

[eig](#) | [det](#) | [rank](#)

**Purpose** Inverse  $z$ -transform

**Syntax**

$$f = \text{iztrans}(F)$$

$$f = \text{iztrans}(F, k)$$

$$f = \text{iztrans}(F, w, k)$$

**Description**  $f = \text{iztrans}(F)$  is the inverse  $z$ -transform of the scalar symbolic object  $F$  with default independent variable  $z$ . The default return is a function of  $n$ .

$$f(n) = \frac{1}{2\pi i} \oint_{|z|=R} F(z)z^{n-1} dz, \quad n = 1, 2, \dots$$

where  $R$  is a positive number chosen so that the function  $F(z)$  is analytic on and outside the circle  $|z| = R$ .

If  $F = F(n)$ ,  $\text{iztrans}$  returns a function of  $k$ .

$$f = f(k)$$

$f = \text{iztrans}(F, k)$  makes  $f$  a function of  $k$  instead of the default  $n$ . Here  $k$  is a scalar symbolic object.

$f = \text{iztrans}(F, w, k)$  takes  $F$  to be a function of  $w$  instead of the default  $\text{symvar}(F)$  and returns a function of  $k$ .

$$F = F(w) \Rightarrow f = f(k)$$

## Examples

Inverse Z-Transform	MATLAB Operation
$f(z) = \frac{2z}{(z-2)^2}$ $Z^{-1}[f] = \frac{1}{2\pi i} \oint_{ z =R} f(s)z^{n-1} dz$ $= n2^n$	<pre>syms z f = 2*z/(z-2)^2; iztrans(f)  returns  ans = 2^n + 2^n*(n - 1)</pre>
$g(n) = \frac{n(n+1)}{n^2 + 2n + 1}$ $Z^{-1} g  = \frac{1}{2\pi i} \oint_{ n =R} g(n)n^{k-1} dn$ $= -1^k$	<pre>syms n g = n*(n+1)/(n^2+2*n+1); iztrans(g)  returns  ans = (-1)^k</pre>
$f(z) = \frac{z}{z-a}$ $Z^{-1}[f] = \frac{1}{2\pi i} \oint_{ z =R} f(z)z^{k-1} dz$ $= a^k \text{ if } a \neq 0$	<pre>syms z a k f = z/(z-a); simplify(iztrans(f,k))  returns  ans = piecewise([a &lt;&gt; 0, a^k])</pre>

## See Also

ifourier, ilaplace, ztrans

**Purpose** Compute Jacobian matrix

**Syntax** `jacobian(f, v)`

**Description** `jacobian(f, v)` computes the Jacobian of the scalar or vector `f` with respect to `v`. The  $(i, j)$ -th entry of the result is  $\partial f(i) / \partial v(j)$ . If `f` is scalar, the Jacobian of `f` is the gradient of `f`. If `v` is a scalar, the result equals to `diff(f, v)`.

**Examples** Compute the Jacobians of the following vectors:

```
syms x y z
f = [x*y*z; y; x + z];
v = [x, y, z];
R = jacobian(f, v)
b = jacobian(x + z, v)
```

The result is

```
R =
[ y*z, x*z, x*y]
[ 0, 1, 0]
[ 1, 0, 1]
```

```
b =
[ 1, 0, 1]
```

**See Also** `diff`

# jordan

---

**Purpose** Compute Jordan canonical form of matrix

**Syntax**  
 $J = \text{jordan}(A)$   
 $[V, J] = \text{jordan}(A)$

**Description**  $J = \text{jordan}(A)$  computes the Jordan canonical (normal) form of a symbolic or numeric matrix  $A$ . The Jordan form of a numeric matrix is extremely sensitive to numerical errors. To compute Jordan canonical form of a matrix, represent the elements of the matrix by integers or ratios of small integers, if possible.

$[V, J] = \text{jordan}(A)$  computes the Jordan canonical form  $J$  and the similarity transform  $V$ . The matrix  $V$  contains the generalized eigenvectors of  $A$  as columns, and  $V \backslash A * V = J$ .

**Examples** Compute the Jordan canonical form and the similarity transform for the following numeric matrix. Verify that the resulting matrix  $V$  satisfies the condition  $V \backslash A * V = J$ :

```
A = [1 -3 -2; -1 1 -1; 2 4 5]
[V, J] = jordan(A)
V \ A * V
```

The result is:

```
A =
     1     -3     -2
    -1      1     -1
     2      4      5
```

```
V =
    -1      1     -1
    -1      0      0
     2      0      1
```

```
J =
     2      1      0
```

```
0 2 0
0 0 3
```

```
ans =
2 1 0
0 2 0
0 0 3
```

**See Also** eig | inv | poly

# lambertw

---

**Purpose** Lambert's  $W$  function

**Syntax**  
`W = lambertw(X)`  
`W = lambertw(K,X)`

**Description** `W = lambertw(X)` evaluates Lambert's  $W$  function at the elements of  $X$ , a numeric matrix or a symbolic matrix. Lambert's  $W$  solves the equation

$$we^w = x$$

for  $w$  as a function of  $x$ .

`W = lambertw(K,X)` is the  $K$ -th branch of this multi-valued function.

**Examples** Compute Lambert's  $W$  function:

```
lambertw([0 -exp(-1); pi 1])
```

The result is:

```
ans =  
      0   -1.0000  
 1.0737   0.5671
```

The statements

```
syms x y  
lambertw([0 x;1 y])
```

return

```
ans =  
[          0, lambertw(0, x)]  
[ lambertw(0, 1), lambertw(0, y)]
```



**References**

[1] Corless, R.M, G.H. Gonnet, D.E.G. Hare, and D.J. Jeffrey, *Lambert's W Function in Maple*, Technical Report, Dept. of Applied Math., Univ. of Western Ontario, London, Ontario, Canada.

[2] Corless, R.M, Gonnet, G.H. Gonnet, D.E.G. Hare, and D.J. Jeffrey, *On Lambert's W Function*, Technical Report, Dept. of Applied Math., Univ. of Western Ontario, London, Ontario, Canada.

Both papers are available by anonymous FTP from

`cs-archive.uwaterloo.ca`

# laplace

---

**Purpose** Laplace transform

**Syntax**  
laplace(F)  
laplace(F, t)  
laplace(F, w, z)

**Description**  $L = \text{laplace}(F)$  is the Laplace transform of the scalar symbol  $F$  with default independent variable  $t$ . The default return is a function of  $s$ . The Laplace transform is applied to a function of  $t$  and returns a function of  $s$ .

$$F = F(t) \Rightarrow L = L(s)$$

If  $F = F(s)$ , `laplace` returns a function of  $t$ .

$$L = L(t)$$

By definition

$$L(s) = \int_0^{\infty} F(t)e^{-st} dt$$

where  $t$  is the symbolic variable in  $F$  as determined by `symvar`.

$L = \text{laplace}(F, t)$  makes  $L$  a function of  $t$  instead of the default  $s$ .

$$L(t) = \int_0^{\infty} F(x)e^{-tx} dx$$

Here  $L$  is returned as a scalar symbol.

$L = \text{laplace}(F, w, z)$  makes  $L$  a function of  $z$  and  $F$  a function of  $w$  instead of the default variables  $s$  and  $t$ , respectively.

$$L(z) = \int_0^{\infty} F(w)e^{-zw} dw$$

## Examples

Laplace Transform	MATLAB Command
$f(t) = t^4$ $L[f] = \int_0^{\infty} f(t)e^{-ts} dt$ $= \frac{24}{s^5}$	<pre>syms t; f = t^4; laplace(f)</pre> <p>returns</p> <p>ans = 24/s^5</p>
$g(s) = \frac{1}{\sqrt{s}}$ $L[g](t) = \int_0^{\infty} g(s)e^{-st} ds$ $= \sqrt{\frac{\pi}{t}}$	<pre>syms s; g = 1/sqrt(s); laplace(g)</pre> <p>returns</p> <p>ans = pi^(1/2)/t^(1/2)</p>
$f(t) = e^{-at}$ $L[f](x) = \int_0^{\infty} f(t)e^{-tx} dt$ $= \frac{1}{x+a}$	<pre>syms t a x; f = exp(-a*t); laplace(f,x)</pre> <p>returns</p> <p>ans = 1/(a + x)</p>

# laplace

---

## **See Also**

fourier, ilaplace, ztrans

<b>Purpose</b>	LaTeX representation of symbolic expression
<b>Syntax</b>	latex(S)
<b>Description</b>	latex(S) returns the LaTeX representation of the symbolic expression S.
<b>Examples</b>	The statements

```

syms x
f = taylor(log(1+x));
latex(f)

return

ans =
\frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x

```

The statements

```

H = sym(hilb(3));
latex(H)

return

ans =
\left(\begin{array}{ccc} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{array}\right)

```

The statements

```

syms t;
alpha = sym('alpha');
A = [alpha t alpha*t];
latex(A)

```

```
return
```

```
ans =  
\left(\begin{array}{ccc} \mathrm{\alpha} & t & \mathrm{\alpha}\backslash, t...  
\end{array}\right)
```

You can use the `latex` command to annotate graphs:

```
syms x  
f = taylor(log(1+x));  
ezplot(f)  
hold on  
title(['$' latex(f) '$'],'interpreter','latex')  
hold off
```

## See Also

`pretty`, `ccode`, `fortran`

**Purpose**

Compute limit of symbolic expression

**Syntax**

```
limit(expr, x, a)
limit(expr, a)
limit(expr)
limit(expr, x, a, 'left')
limit(expr, x, a, 'right')
```

**Description**

`limit(expr, x, a)` computes bidirectional limit of the symbolic expression `expr` when `x` approaches `a`.

`limit(expr, a)` computes bidirectional limit of the symbolic expression `expr` when the default variable approaches `a`.

`limit(expr)` computes bidirectional limit of the symbolic expression `expr` when the default variable approaches 0.

`limit(expr, x, a, 'left')` computes the limit of the symbolic expression `expr` when `x` approaches `a` from the left.

`limit(expr, x, a, 'right')` computes the limit of the symbolic expression `expr` when `x` approaches `a` from the right.

**Examples**

Compute bidirectional limits for the following expressions:

```
syms x h;
limit(sin(x)/x)
limit((sin(x + h) - sin(x))/h, h, 0)
```

The results are

```
ans =
1
```

```
ans =
cos(x)
```

# limit

---

Compute the limits from the left and right for the following expressions:

```
syms x;  
limit(1/x, x, 0, 'right')  
limit(1/x, x, 0, 'left')
```

The results are

```
ans =  
Inf  
  
ans =  
-Inf
```

Compute the limit for the functions presented as elements of a vector:

```
syms x a;  
v = [(1 + a/x)^x, exp(-x)];  
limit(v, x, inf)
```

The result is

```
ans =  
[ exp(a), 0]
```

## See Also

[diff](#) | [taylor](#)



<b>Purpose</b>	Logarithm base 10 of entries of symbolic matrix
<b>Syntax</b>	$Y = \log_{10}(X)$
<b>Description</b>	$Y = \log_{10}(X)$ returns the logarithm to the base 10 of $X$ . If $X$ is a matrix, $Y$ is a matrix of the same size, each entry of which is the logarithm of the corresponding entry of $X$ .
<b>See Also</b>	<code>log2</code>

# log2

---

**Purpose**            Logarithm base 2 of entries of symbolic matrix

**Syntax**             $Y = \log_2(X)$

**Description**         $Y = \log_2(X)$  returns the logarithm to the base 2 of  $X$ . If  $X$  is a matrix,  $Y$  is a matrix of the same size, each entry of which is the logarithm of the corresponding entry of  $X$ .

**See Also**            `log10`

**Purpose**

Convert symbolic expression to function handle or file

**Syntax**

```
g = matlabFunction(f)
g = matlabFunction(f1, f2, ...)
g = matlabFunction(f,param1,value1,...)
```

**Description**

`g = matlabFunction(f)` converts the symbolic expression `f` to a MATLAB function with the handle `g`.

`g = matlabFunction(f1, f2, ...)` converts a list of the symbolic expressions `f1, f2, ...` to a MATLAB function with multiple outputs. The function handle is `g`.

`g = matlabFunction(f,param1,value1,...)` converts the symbolic expression `f` to a MATLAB function with the handle `g`. The command accepts the following options for parameter/value pairs:

- `Parameter = 'file'` allows you to generate an *optimized* M-file that can accept double or matrix arguments and evaluate the symbolic expression applied to the arguments. Optimized means intermediate variables are automatically generated to simplify or speed the code. `value` should be a string representing the path to the M-file. If the string is empty, `matlabFunction` generates an anonymous function. If the string does not end in `.m`, the function appends `.m`.
- `Parameter = 'outputs'` allows you to set the names of the output variables. `value` should be a cell array of strings. The default names of output variables coincide with the names you use calling `matlabFunction`. If you call `matlabFunction` using an expression instead of individual variables, the default names of output variables consist of the word `out` followed by the number, for example, `out3`.
- `Parameter = 'vars'` allows you to set the order of the input variables or symbolic vectors in the resulting function handle or M-file. The default order is alphabetical. `value` should be either a cell array of strings or symbolic arrays, or a vector of symbolic variables. The number of value entries should equal or exceed the number of free variables in the symbolic expression `f`.

# matlabFunction

---

---

**Note** To convert a MuPAD expression or function to a MATLAB function, use `f = evalin(symengine, 'MuPAD_Expression')` or `f = feval(symengine, 'MuPAD_Function', x1, ..., xn)`. `matlabFunction` cannot correctly convert some MuPAD expressions to MATLAB functions. These expressions do not trigger an error message. When converting a MuPAD expression or function that is not on the MATLAB vs. MuPAD Expressions list, always check the results of conversion. To verify the results, execute the resulting function.

---

Symbolic Math Toolbox with a Maple engine does not support `matlabFunction`. For details, see “Differences in Functionality When Using MuPAD and Maple Engines” on page 4-43.

## Examples

```
syms x y
r = sqrt(x^2 + y^2);
ht = matlabFunction(sin(r)/r)

ht =
    @(x,y)sin(sqrt(x.^2+y.^2)).*1./sqrt(x.^2+y.^2)
```

The following example generates a file:

```
syms x y z
r = x^2 + y^2 + z^2;
f = matlabFunction(log(r)+r^(-1/2), 'file', 'myfile');
```

If the file `myfile.m` already exists in the current directory, `matlabFunction` replaces the existing function with the converted symbolic expression. You can open and edit the resulting file:

```
function out1 = myfile(x,y,z)
%MYFILE
%    OUT1 = MYFILE(X,Y,Z)

%    This function was generated
```

```
% by the Symbolic Math Toolbox version 5.2.
% 07-Nov-2008 14:38:59

t2 = x.^2;
t3 = y.^2;
t4 = z.^2;
t5 = t2 + t3 + t4;
out1 = log(t5) + 1./t5.^(1./2);
```

You can change the order of the input variables:

```
syms x y z
r = x^2 + y^2 + z^2;
matlabFunction(r, 'file', 'new_function',...
'vars', [y z x]);
```

The created `new_function` accepts variables in the required order:

```
function r = new_function(y,z,x)
%NEW_FUNCTION
% R = NEW_FUNCTION(Y,Z,X)

% This function was generated
% by the Symbolic Math Toolbox version 5.2.
% 31-Oct-2008 18:13:19

r = x.^2 + y.^2 + z.^2;
```

You can specify that the input arguments are vectors:

```
syms x y z t
r = x^2 + y^2 + z^2;
matlabFunction(r, 'file', 'new_function',...
'vars', {t, [x y z]});
```

The resulting function operates on vectors:

```
function r = new_function(t,in2)
```

# matlabFunction

---

```
%NEW_FUNCTION
%   R = NEW_FUNCTION(T,IN2)

%   This function was generated
%   by the Symbolic Math Toolbox version 5.2.
%   03-Nov-2008 18:19:37

x = in2(:,1);
y = in2(:,2);
z = in2(:,3);
r = x.^2 + y.^2 + z.^2;
```

You can name of the output variables:

```
syms x y z
r = x^2 + y^2 + z^2;
q = x^2 - y^2 - z^2;
f = matlabFunction(r, q, 'file', 'new_function',...
'outputs', {'name1','name2'});
```

The generated function returns *name1* and *name2*:

```
function [name1,name2] = new_function(x,y,z)
%NEW_FUNCTION
%   [NAME1,NAME2] = NEW_FUNCTION(X,Y,Z)

%   This function was generated
%   by the Symbolic Math Toolbox version 5.2.
%   31-Oct-2008 18:20:48

t9 = x.^2;
t10 = y.^2;
t11 = z.^2;
name1 = t10 + t11 + t9;
if nargin > 1
    name2 = t9 - t11 - t10;
end
```

Also, you can convert MuPAD expressions:

```
syms x y;  
f = evalin(symengine, 'arcsin(x) + arccos(y)');  
matlabFunction(f, 'file', 'new_function');
```

The created file contains the same expressions written in the MATLAB language:

```
function f = new_function(x,y)  
%NEW_FUNCTION  
%   F = NEW_FUNCTION(X,Y)  
  
%   This function was generated  
%   by the Symbolic Math Toolbox version 5.2.  
%   31-Oct-2008 17:41:12  
  
f = asin(x) + acos(y);
```

## See Also

`ccode`, `fortran`, `subs`, `sym2poly`, `emlBlock`

“Generating Code from Symbolic Expressions” on page 3-129

# mfun

---

**Purpose** Numeric evaluation of special mathematical function

**Syntax** `mfun('function',par1,par2,par3,par4)`

**Description** `mfun('function',par1,par2,par3,par4)` numerically evaluates one of the Maple special mathematical functions. If you use the MuPAD engine, Symbolic Math Toolbox software translates the name of the Maple function to a call to the appropriate MuPAD functions. If you use the Maple engine, the toolbox calls the Maple functions directly. Each `par` argument is a numeric quantity corresponding to a parameter for `function`. You can use up to four parameters. The last parameter specified can be a matrix, usually corresponding to  $X$ . The dimensions of all other parameters depend on the specifications for `function`. You can access parameter information for `mfun` functions in “Syntax and Definitions of `mfun` Special Functions” on page 6-129.

MuPAD software evaluates `function` using 16-digit accuracy. Each element of the result is a MATLAB numeric quantity. Any singularity in `function` is returned as NaN.

**Examples** Evaluate Fresnel cosine integral:

```
mfun('FresnelC',0:5)
```

The result is:

```
ans =  
0    0.7799    0.4883    0.6057    0.4984    0.5636
```

Evaluate hyperbolic cosine integral:

```
mfun('Chi',[3*i 0])
```

```
ans =  
0.1196 + 1.5708i    NaN
```

**See Also** `mfunlist`



**Purpose** List special functions for use with mfun

**Syntax** mfunlist

**Description** mfunlist lists the special mathematical functions for use with the mfun function. The following tables describe these special functions.

**Syntax and Definitions of mfun Special Functions** The following conventions are used in the next table, unless otherwise indicated in the **Arguments** column.

x, y	real argument
z, z1, z2	complex argument
m, n	integer argument

**MFUN Special Functions**

Function Name	Definition	mfun Name	Arguments
Bernoulli numbers and polynomials	Generating functions: $\frac{e^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \cdot \frac{t^{n-1}}{n!}$	bernoulli(n) bernoulli(n,t)	$n \geq 0$ $0 <  t  < 2\pi$
Bessel functions	BesselI, BesselJ—Bessel functions of the first kind. BesselK, BesselY—Bessel functions of the second kind.	BesselJ(v,x) BesselY(v,x) BesselI(v,x) BesselK(v,x)	v is real.
Beta function	$B(x,y) = \frac{\Gamma(x) \cdot \Gamma(y)}{\Gamma(x+y)}$	Beta(x,y)	

## MFUN Special Functions (Continued)

Function Name	Definition	mfun Name	Arguments
Binomial coefficients	$\binom{m}{n} = \frac{m!}{n!(m-n)!}$ $= \frac{\Gamma(m+1)}{\Gamma(n+1)\Gamma(m-n+1)}$	binomial(m,n)	
Complete elliptic integrals	Legendre's complete elliptic integrals of the first, second, and third kind	EllipticK(k) EllipticE(k) EllipticPi(a,k)	a is real, $-\infty < a < \infty$ . k is real, $0 < k < 1$ .
Complete elliptic integrals with complementary modulus	Associated complete elliptic integrals of the first, second, and third kind using complementary modulus	EllipticCK(k) EllipticCE(k) EllipticCPi(a,k)	a is real, $-\infty < a < \infty$ . k is real, $0 < k < 1$ .
Complementary error function and its iterated integrals	$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \cdot \int_z^{\infty} e^{-t^2} dt = 1 - \operatorname{erf}(z)$ $\operatorname{erfc}(-1, z) = \frac{2}{\sqrt{\pi}} \cdot e^{-z^2}$ $\operatorname{erfc}(n, z) = \int_z^{\infty} \operatorname{erfc}(n-1, t) dt$	erfc(z) erfc(n, z)	$n > 0$
Dawson's integral	$F(x) = e^{-x^2} \cdot \int_0^x e^{t^2} dt$	dawson(x)	

**MFUN Special Functions (Continued)**

<b>Function Name</b>	<b>Definition</b>	<b>mfun Name</b>	<b>Arguments</b>
Digamma function	$\Psi(x) = \frac{d}{dx} \ln(\Gamma(x)) = \frac{\Gamma'(x)}{\Gamma(x)}$	Psi(x)	
Dilogarithm integral	$f(x) = \int_1^x \frac{\ln(t)}{1-t} dt$	dilog(x)	$x > 1$
Error function	$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$	erf(z)	
Euler numbers and polynomials	Generating function for Euler numbers: $\frac{1}{\cosh(t)} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}$	euler(n) euler(n,z)	$n \geq 0$ $ t  < \frac{\pi}{2}$
Exponential integrals	$Ei(n,z) = \int_1^{\infty} \frac{e^{-zt}}{t^n} dt$ $Ei(x) = PV \left( - \int_{-\infty}^x \frac{e^t}{t} \right)$	Ei(n,z) Ei(x)	$n \geq 0$ Real(z) > 0
Fresnel sine and cosine integrals	$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$ $S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$	FresnelC(x) FresnelS(x)	

## MFUN Special Functions (Continued)

Function Name	Definition	mfun Name	Arguments
Gamma function	$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$	GAMMA(z)	
Harmonic function	$h(n) = \sum_{k=1}^n \frac{1}{k} = \Psi(n+1) + \gamma$	harmonic(n)	$n > 0$
Hyperbolic sine and cosine integrals	$Shi(z) = \int_0^z \frac{\sinh(t)}{t} dt$ $Chi(z) = \gamma + \ln(z) + \int_0^z \frac{\cosh(t) - 1}{t} dt$	Shi(z) Chi(z)	
(Generalized) hypergeometric function	$F(n, d, z) = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^j \frac{\Gamma(n_i + k)}{\Gamma(n_i)} \cdot z^k}{\prod_{i=1}^m \frac{\Gamma(d_i + k)}{\Gamma(d_i)} \cdot k!}$ <p>where <math>j</math> and <math>m</math> are the number of terms in <math>n</math> and <math>d</math>, respectively.</p>	hypergeom(n, d, x) where $n = [n_1, n_2, \dots]$ $d = [d_1, d_2, \dots]$	$n_1, n_2, \dots$ are real. $d_1, d_2, \dots$ are real and nonnegative.
Incomplete elliptic integrals	Legendre's incomplete elliptic integrals of the first, second, and third kind.	EllipticF(x, k) EllipticE(x, k) EllipticPi(x, a, k)	$0 < x \leq \infty$ . $a$ is real, $-\infty < a < \infty$ . $k$ is real, $0 < k < 1$ .

**MFUN Special Functions (Continued)**

<b>Function Name</b>	<b>Definition</b>	<b>mfun Name</b>	<b>Arguments</b>
Incomplete gamma function	$\Gamma(a, z) = \int_z^{\infty} e^{-t} \cdot t^{a-1} dt$	GAMMA(z1, z2) z1 = a z2 = z	
Logarithm of the gamma function	$\ln\text{GAMMA}(z) = \ln(\Gamma(z))$	lnGAMMA(z)	
Logarithmic integral	$Li(x) = PV \left\{ \int_0^x \frac{dt}{\ln t} \right\} = Ei(\ln x)$	Li(x)	x > 1
Polygamma function	$\Psi^{(n)}(z) = \frac{d^n}{dz} \Psi(z)$ where $\Psi(z)$ is the Digamma function.	Psi(n, z)	n ≥ 0
Shifted sine integral	$Ssi(z) = Si(z) - \frac{\pi}{2}$	Ssi(z)	

The following orthogonal polynomials are available using mfun. In all cases, n is a nonnegative integer and x is real.

**Orthogonal Polynomials**

<b>Polynomial</b>	<b>mfun Name</b>	<b>Arguments</b>
Chebyshev of the first and second kind	T(n, x) U(n, x)	

## Orthogonal Polynomials (Continued)

Polynomial	mfun Name	Arguments
Gegenbauer	$G(n, a, x)$	$a$ is a nonrational algebraic expression or a rational number greater than $-1/2$ .
Hermite	$H(n, x)$	
Jacobi	$P(n, a, b, x)$	$a, b$ are nonrational algebraic expressions or rational numbers greater than $-1$ .
Laguerre	$L(n, x)$	
Generalized Laguerre	$L(n, a, x)$	$a$ is a nonrational algebraic expression or a rational number greater than $-1$ .
Legendre	$P(n, x)$	

### Examples

```
mfun('H',5,10)
```

```
ans =  
    3041200
```

```
mfun('dawson',3.2)
```

```
ans =  
    0.1655
```

### Limitations

In general, the accuracy of a function will be lower near its roots and when its arguments are relatively large.

Running time depends on the specific function and its parameters. In general, calculations are slower than standard MATLAB calculations.

**See Also**

mfun

**References**

[1] Abramowitz, M. and I.A., Stegun, *Handbook of Mathematical Functions*, Dover Publications, 1965.

# mod

---

**Purpose** Symbolic matrix elementwise modulus

**Syntax**  $C = \text{mod}(A, B)$

**Description**  $C = \text{mod}(A, B)$  for symbolic matrices  $A$  and  $B$  with integer elements is the positive remainder in the elementwise division of  $A$  by  $B$ . For matrices with polynomial entries,  $\text{mod}(A, B)$  is applied to the individual coefficients.

## Examples

```
ten = sym('10');  
mod(2^ten, ten^3)
```

```
ans =  
24
```

```
syms x  
mod(x^3 - 2*x + 999, 10)
```

```
ans =  
x^3 + 8*x + 9
```

**See Also** `quorem`



---

<b>Purpose</b>	Start MuPAD notebook
<b>Syntax</b>	<pre>mphandle = mupad mphandle = mupad(file)</pre>
<b>Description</b>	<p><code>mphandle = mupad</code> creates a MuPAD notebook, and keeps a handle (pointer) to the notebook in the variable <code>mphandle</code>. You can use any variable name you like instead of <code>mphandle</code>.</p> <p><code>mphandle = mupad(file)</code> opens the MuPAD notebook or program file named <code>file</code>, and keeps a handle (pointer) to the notebook or program file in the variable <code>mphandle</code>. This syntax has the functionality of both <code>openmn</code> and <code>openmu</code>.</p>
<b>Examples</b>	<p>To start a new notebook and define a handle <code>mphandle</code> to the notebook, enter:</p> <pre>mphandle = mupad;</pre> <p>To open an existing notebook named <code>notebook1.mn</code> located in the current directory, and define a handle <code>mphandle</code> to the notebook, enter:</p> <pre>mphandle = mupad('notebook1.mn');</pre>
<b>See Also</b>	<code>getVar</code> , <code>mupadwelcome</code> , <code>openmn</code> , <code>openmu</code> , <code>setVar</code>

# mupadwelcome

---

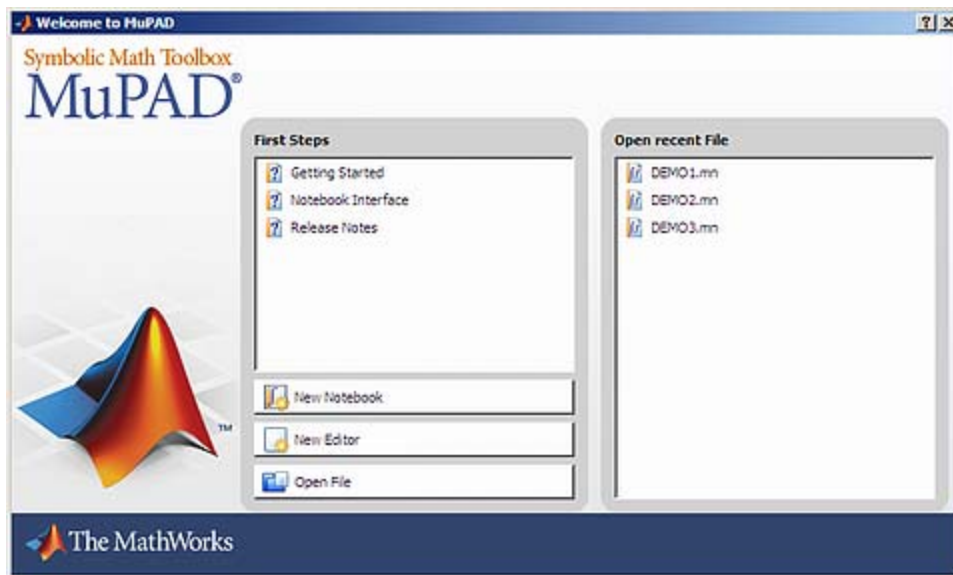
**Purpose** Launch MuPAD interfaces

**Syntax** mupadwelcome

**Description** mupadwelcome brings up a window that enables you to launch various MuPAD interfaces:

- Notebook, for performing calculations
- Editor, for writing programs and libraries
- Help, in the **First Steps** pane

It also enables you to access recent MuPAD files or browse for files.



**See Also** mupad

“Opening Recent Files and Other MuPAD Interfaces” on page 4-13

**Purpose**

Form a basis for null space of matrix

**Syntax**

`Z = null(A)`

**Description**

`Z = null(A)` returns a list of vectors that form the basis for the null space of a matrix `A`. The product `A*Z` is zero. `size(Z, 2)` is the nullity of `A`. If `A` has full rank, `Z` is empty.

**Examples**

Find the basis for the null space and the nullity of the magic square of symbolic numbers. Verify that `A*Z` is zero:

```
A = sym(magic(4));
Z = null(A)
nullityOfA = size(Z, 2)
A*Z
```

The results are:

```
Z =
-1
-3
 3
 1

nullityOfA =
 1

ans =
 0
 0
 0
 0
```

---

Find the basis for the null space of the matrix `B` that has full rank:

# null

---

```
B = sym(hilb(3))
Z = null(B)
```

The result is:

```
B =
[ 1, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]

Z =
[ empty sym ]
```

## See Also

[rank](#) | [rref](#) | [size](#) | [svd](#)

**Purpose** Numerator and denominator

**Syntax** `[N,D] = numden(A)`

**Description** `[N,D] = numden(A)` converts each element of `A` to a rational form where the numerator and denominator are relatively prime polynomials with integer coefficients. `A` is a symbolic or a numeric matrix. `N` is the symbolic matrix of numerators, and `D` is the symbolic matrix of denominators.

**Examples** Find the numerator and denominator of the symbolic number:

```
[n, d] = numden(sym(4/5))
```

The result is:

```
n =  
4
```

```
d =  
5
```

Find the numerator and denominator of the symbolic expression:

```
syms x y;  
[n,d] = numden(x/y + y/x)
```

The result is:

```
n =  
x^2 + y^2
```

```
d =  
x*y
```

The statements

# numden

---

```
syms a b
A = [a, 1/b]
[n,d] = numden(A)

return

A =
[a, 1/b]

n =
[a, 1]

d =
[1, b]
```

<b>Purpose</b>	Open MuPAD notebook
<b>Syntax</b>	<code>nb = openmn(file)</code>
<b>Description</b>	<code>nb = openmn(file)</code> opens the MuPAD notebook <code>file</code> , and returns a handle to the file in <code>nb</code> . <code>nb = mupad(file)</code> accomplishes the same task.
<b>Example</b>	To open a notebook named <code>e-e-x.mn</code> in the directory <code>\Documents\Notes 2009b</code> of drive <code>H:</code> , enter <pre>nb = openmn('H:\Documents\Notes 2009b\e-e-x.mn');</pre>
<b>See Also</b>	<code>mupad</code> , <code>openmu</code>

# openmu

---

**Purpose** Open MuPAD program file

**Syntax** `nb = openmu(file)`

**Description** `nb = openmu(file)` opens the MuPAD program file, and returns a handle to the file in `nb`. `nb = mupad(file)` accomplishes the same task.

**Example** To open a program file named `yyx.mu` in the directory `\Documents\Notes 2009b` of drive `H:`, enter

```
nb = openmu('H:\Documents\Notes 2009b\yyx.mu');
```

**See Also** `mupad`, `openmn`



**Purpose**

Compute characteristic polynomial of matrix

**Syntax**

```
p = poly(A)
p = poly(A, v)
poly(sym(A))
```

**Description**

`p = poly(A)` returns the coefficients of the characteristic polynomial of a numeric matrix `A`. For symbolic `A`, `poly(A)` returns the characteristic polynomial of `A` in terms of the default variable `x`. If the elements of `A` already contain the variable `x`, the default variable is `t`. If the elements of `A` contain both `x` and `t`, the default variable is still `t`.

`p = poly(A, v)`, for both numeric and symbolic matrices, returns the characteristic polynomial of `A` in terms of the variable `v`.

`poly(sym(A))`, for numeric `A`, approximately equals `poly2sym(poly(A))`. The approximation is due to roundoff error.

**Examples**

Compute characteristic polynomials of one of the MATLAB test matrices:

```
syms z
A = gallery(3)
p = poly(A)
q = poly(sym(A))
s = poly(A, z)
```

The results are:

```
A =
   -149   -50  -154
    537   180   546
    -27    -9   -25

p =
    1.0000   -6.0000   11.0000   -6.0000
```

```
q =  
x^3 - 6*x^2 + 11*x - 6
```

```
s =  
z^3 - 6*z^2 + 11*z - 6
```

---

Compute the characteristic polynomials of the following symbolic matrix in terms of the default variable. Also compute the characteristic polynomials in terms of the specified variable *y*:

```
syms x y;  
B = x*hilb(3)  
a = poly(B)  
b = poly(B, y)
```

The results are:

```
B =  
[ x, x/2, x/3]  
[ x/2, x/3, x/4]  
[ x/3, x/4, x/5]
```

```
a =  
t^3 - (23*t^2*x)/15 + (127*t*x^2)/720 - x^3/2160
```

```
b =  
- x^3/2160 + (127*x^2*y)/720 - (23*x*y^2)/15 + y^3
```

## See Also

[eig](#) | [jordan](#) | [poly2sym](#) | [solve](#)

**Purpose** Polynomial coefficient vector to symbolic polynomial

**Syntax**  
`r = poly2sym(c)`  
`r = poly2sym(c, v)`

**Description** `r = poly2sym(c)` returns a symbolic representation of the polynomial whose coefficients are in the numeric vector `c`. The default symbolic variable is `x`. The variable `v` can be specified as a second input argument. If `c = [c1 c2 ... cn]`, `r = poly2sym(c)` has the form

$$c_1x^{n-1} + c_2x^{n-2} + \dots + c_n$$

`poly2sym` uses `sym`'s default (rational) conversion mode to convert the numeric coefficients to symbolic constants. This mode expresses the symbolic coefficient approximately as a ratio of integers, if `sym` can find a simple ratio that approximates the numeric value, otherwise as an integer multiplied by a power of 2.

`r = poly2sym(c, v)` is a polynomial in the symbolic variable `v` with coefficients from the vector `c`. If `v` has a numeric value and `sym` expresses the elements of `c` exactly, `eval(poly2sym(c))` returns the same value as `polyval(c, v)`.

**Examples** The command

```
poly2sym([1 3 2])
```

returns

```
ans =
x^2 + 3*x + 2
```

The command

```
poly2sym([.694228, .333, 6.2832])
```

returns

# poly2sym

---

```
ans =  
(6253049924220329*x^2)/9007199254740992 + (333*x)/1000 + 3927/625
```

The command

```
poly2sym([1 0 1 -1 2], y)
```

returns

```
ans =  
y^4 + y^2 - y + 2
```

## See Also

`sym`, `sym2poly`, `polyval` in the online MATLAB Function Reference

**Purpose** Pretty-print symbolic expressions

**Syntax**

**Description** The pretty function prints symbolic output in a format that resembles typeset mathematics.

**Examples** The following statements

```
A = sym(pascal(2))
B = eig(A)
pretty(B)
```

return

```
A =
[ 1, 1]
[ 1, 2]

B =
3/2 - 5^(1/2)/2
5^(1/2)/2 + 3/2
```

```
+ -          - +
|           1/2 |
|  3   5     |
|  - - ----  |
|  2   2     |
|           |
|   1/2      |
|  5       3  |
|  ---- + -  |
|  2       2  |
+ -          - +
```

# quorem

---

**Purpose** Symbolic matrix elementwise quotient and remainder

**Syntax**  $[Q, R] = \text{quorem}(A, B)$

**Description**  $[Q, R] = \text{quorem}(A, B)$  for symbolic matrices  $A$  and  $B$  with integer or polynomial elements does elementwise division of  $A$  by  $B$  and returns quotient  $Q$  and remainder  $R$  so that  $A = Q.*B+R$ . For polynomials,  $\text{quorem}(A,B,x)$  uses variable  $x$  instead of  $\text{symvar}(A,1)$  or  $\text{symvar}(B,1)$ .

## Example

```
syms x
p = x^3 - 2*x + 5;
[q, r] = quorem(x^5, p)

q =
x^2 + 2

r =
- 5*x^2 + 4*x - 10

[q, r] = quorem(10^5, subs(p,'10'))

q = 101
r = 515
```

**See Also** `mod`

---

<b>Purpose</b>	Compute the rank of symbolic matrix
<b>Syntax</b>	<code>rank(A)</code>
<b>Description</b>	<code>rank(A)</code> computes the rank of the symbolic matrix A.
<b>Examples</b>	Compute the rank of the following numeric matrix:

```
B = magic(4);  
rank(B)
```

The result is:

```
ans =  
    3
```

---

Compute the rank of the following symbolic matrix:

```
syms a b c d  
A = [a b;c d];  
rank(A)
```

The result is:

```
ans =  
    2
```

**See Also** [eig](#) | [null](#) | [rref](#) | [size](#)

# real

---

**Purpose** Real part of a complex symbolic number

**Syntax** `real(Z)`

**Description** `real(Z)` is the real part of a symbolic `Z`.

**See Also** `conj`, `imag`



**Purpose** Close MuPAD engine

**Syntax** `reset(symengine)`

**Description** `reset(symengine)` closes the MuPAD engine associated with the MATLAB workspace, and resets all its assumptions. Immediately before or after executing `reset(symengine)` you should clear all symbolic objects in the MATLAB workspace.

**See Also** `symengine`

# round

---

**Purpose** Symbolic matrix elementwise round

**Syntax**  $Y = \text{round}(X)$

**Description**  $Y = \text{round}(X)$  rounds the elements of  $X$  to the nearest integers. Values halfway between two integers are rounded away from zero.

**Example**

```
x = sym(-5/2);  
[fix(x) floor(x) round(x) ceil(x) frac(x)]
```

```
ans =  
[ -2, -3, -3, -2, -1/2]
```

**See Also** floor, ceil, fix, frac

**Purpose** Compute reduced row echelon form of matrix

**Syntax** `rref(A)`

**Description** `rref(A)` computes the reduced row echelon form of the symbolic matrix `A`. If the elements of a matrix contain free symbolic variables, `rref` regards the matrix as nonzero.

**Examples** Compute the reduced row echelon form of the magic square matrix:

```
rref(sym(magic(4)))
```

The result is:

```
ans =  
[ 1, 0, 0, 1]  
[ 0, 1, 0, 3]  
[ 0, 0, 1, -3]  
[ 0, 0, 0, 0]
```

---

Compute the reduced row echelon form of the following symbolic matrix:

```
syms a b c;  
A = [a b c; b c a; a + b, b + c, c + a];  
rref(A)
```

The result is:

```
ans =  
[ 1, 0, -(a*b - c^2)/(a*c - b^2)]  
[ 0, 1, -(b*c - a^2)/(a*c - b^2)]  
[ 0, 0, 0]
```

**See Also** `eig` | `jordan` | `rank` | `size`

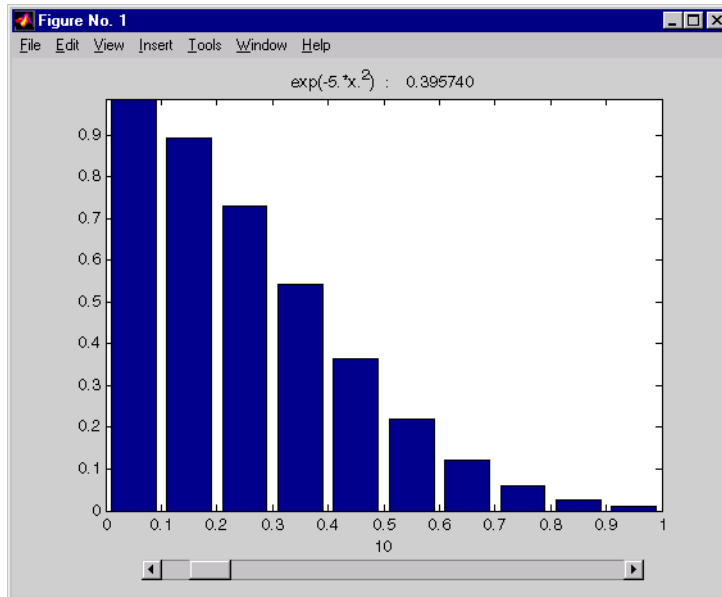
**Purpose** Interactive evaluation of Riemann sums

**Syntax**  
`rsums(f)`  
`rsums(f, a, b)`  
`rsums(f, [a, b])`

**Description** `rsums(f)` interactively approximates the integral of  $f(x)$  by Riemann sums from 0 to 1. `rsums(f)` displays a graph of  $f(x)$ . You can then adjust the number of terms taken in the Riemann sum by using the slider below the graph. The number of terms available ranges from 2 to 128. `f` can be a string or a symbolic expression.

`rsums(f, a, b)` and `rsums(f, [a, b])` approximates the integral from `a` to `b`.

**Examples** Both `rsums('exp(-5*x^2)')` and `rsums exp(-5*x^2)` create the following plot.



**Purpose**

Assign variable in MuPAD notebook

**Syntax**

```
setVar(nb, y)
setVar(nb, 'v', y)
```

**Description**

`setVar(nb, y)` assigns the symbolic expression `y` in the MATLAB workspace to the variable `y` in the MuPAD notebook `nb`.

`setVar(nb, 'v', y)` assigns the symbolic expression `y` in the MATLAB workspace to the variable `v` in the MuPAD notebook `nb`.

**Examples**

```
mpnb = mupad;
syms x;
y = exp(-x);
setVar(mpnb,y)
setVar(mpnb,'z',sin(y))
```

After executing these statements, the MuPAD engine associated with the `mpnb` notebook contains the variables `y`, with value `exp(-x)`, and `z`, with value `sin(exp(-x))`.

**See Also**

`getVar`, `mupad`

# simple

---

**Purpose** Search for simplest form of symbolic expression

**Syntax** `r = simple(S)`  
`[r, how] = simple(S)`

**Description** `r = simple(S)` tries several different algebraic simplifications of the symbolic expression `S`, displays any that shorten the length of `S`'s representation, and returns the shortest. `S` is a `sym`. If `S` is a matrix, the result represents the shortest representation of the entire matrix, which is not necessarily the shortest representation of each individual element. If no return output is given, `simple(S)` displays all possible representations and returns the shortest.

`[r, how] = simple(S)` does not display intermediate simplifications, but returns the shortest found, as well as a string describing the particular simplification. `r` is a `sym`. `how` is a string.

**Examples** Simplify the expressions:

```
syms x;  
f = cos(x)^2 + sin(x)^2;  
f = simple(f)  
g = cos(3*acos(x));  
g = simple(g)
```

The results are:

```
f =  
1  
  
g =  
4*x^3 - 3*x
```

Simplify the expressions displaying all possible simplifications:

```
syms x;  
f = cos(x) + i*sin(x);
```

```
simple(f)
```

The result is:

```
simplify:  
cos(x) + i*sin(x)
```

```
radsimp:  
cos(x) + i*sin(x)
```

```
simplify(100):  
cos(x) + i*sin(x)
```

```
combine(sincos):  
cos(x) + i*sin(x)
```

```
combine(sinhcosh):  
cos(x) + i*sin(x)
```

```
combine(ln):  
cos(x) + i*sin(x)
```

```
factor:  
cos(x) + i*sin(x)
```

```
expand:  
cos(x) + i*sin(x)
```

```
combine:  
cos(x) + i*sin(x)
```

```
rewrite(exp):  
exp(i*x)
```

```
rewrite(sincos):  
cos(x) + i*sin(x)
```

# simple

---

```
rewrite(sinhcosh):
cosh(-i*x) - sinh(-i*x)

rewrite(tan):
(2*i*tan(x/2))/(tan(x/2)^2 + 1) - (tan(x/2)^2 - 1)/(tan(x/2)^2 + 1)

mwcossin:
- 2*sin(x/2)^2 + i*sin(x) + 1

collect(x):
cos(x) + i*sin(x)

ans =
exp(i*x)
```

Simplify the expression and display the simplification method:

```
syms x;
f = (x + 1)*x*(x - 1);
[f, how] = simple(f)

f =
x^3 - x

how =
simplify(100)
```

## See Also

`collect`, `expand`, `factor`, `horner`, `simplify`



**Purpose** Symbolic simplification

**Syntax** `R = simplify(S)`  
`R = simplify(S, n)`

**Description** `R = simplify(S)` simplifies each element of the symbolic matrix `S` using MuPAD simplification rules.

`R = simplify(S, n)` uses the positive integer `n` to control how many steps of simplification the `simplify` function attempts. The default value (without `n`) is `n = 100`.

**Examples** Simplify the trigonometric expression:

```
syms x;
simplify(sin(x)^2 + cos(x)^2)
```

The result is:

```
ans =
1
```

Simplify the expression:

```
syms a b c;
simplify(exp(c*log(sqrt(a+b))))
```

The result is:

```
ans =
(a + b)^(c/2)
```

Simplify the expressions from the list:

```
S = [(x^2 + 5*x + 6)/(x + 2), sqrt(16)];
R = simplify(S)
```

The result is:

# simplify

---

```
R =  
[ x + 3, 4]
```

**See Also**      collect, expand, factor, horner, simple

**Purpose** Convert symbolic matrix to single precision

**Syntax** `single(S)`

**Description** `single(S)` converts the symbolic matrix `S` to a matrix of single-precision floating-point numbers. `S` must not contain any symbolic variables, except `'eps'`.

**See Also** `sym`, `vpa`, `double`

# sinint

---

**Purpose** Sine integral

**Syntax** `Y = sinint(X)`

**Description** `Y = sinint(X)` evaluates the sine integral function at the elements of `X`, a numeric matrix, or a symbolic matrix. The result is a numeric matrix. The sine integral function is defined by

$$Si(x) = \int_0^x \frac{\sin t}{t} dt$$

**Examples** Evaluate sine integral for the elements of the matrix:

```
sinint([pi 0;-2.2 exp(3)])
```

```
ans =  
    1.8519         0  
   -1.6876    1.5522
```

The statement

```
sinint(1.2)
```

returns

```
ans =  
    1.1080
```

The statement

```
syms x;  
diff(sinint(x))
```

returns

```
ans =
```

`sin(x)/x`

**See Also**

`cosint`

# size

---

**Purpose** Symbolic matrix dimensions

**Syntax**

```
d = size(A)
[m, n] = size(A)
d = size(A, n)
```

**Description** Suppose  $A$  is an  $m$ -by- $n$  symbolic or numeric matrix. The statement `d = size(A)` returns a numeric vector with two integer components, `d = [m,n]`.

The multiple assignment statement `[m, n] = size(A)` returns the two integers in two separate variables.

The statement `d = size(A, n)` returns the length of the dimension specified by the scalar  $n$ . For example, `size(A, 1)` is the number of rows of  $A$  and `size(A, 2)` is the number of columns of  $A$ .

**Examples** The statements

```
syms a b c d
A = [a b c ; a b d; d c b; c b a];
d = size(A)
r = size(A, 2)
```

return

```
d =
     4     3
```

```
r =
     3
```

**See Also** `length`, `ndims` in the online MATLAB Function Reference

**Purpose** Symbolic solution of algebraic equations

**Syntax**

```
solve(eq)
solve(eq, var)
solve(eq1, eq2, ..., eqn)
g = solve(eq1, eq2, ..., eqn, var1, var2, ..., varn)
```

**Description** **Single Equation/Expression**

The input to `solve` can be either symbolic expressions or strings. If `eq` is a symbolic expression ( $x^2 - 2x + 1$ ) or a string that does not contain an equal sign (`'x^2 - 2*x + 1'`), then `solve(eq)` solves the equation  $eq = 0$  for its default variable (as determined by `symvar`).

`solve(eq, var)` solves the equation `eq` (or  $eq = 0$  in the two cases cited above) for the variable `var`.

**System of Equations**

The inputs are either symbolic expressions or strings specifying equations. `solve(eq1, eq2, ..., eqn)` or solves the system of equations implied by `eq1, eq2, ..., eqn` in the  $n$  variables determined by applying `symvar` to the system.

`g = solve(eq1, eq2, ..., eqn, var1, var2, ..., varn)` finds the zeros for the system of equations for the variables specified as inputs.

Three different types of output are possible. For one equation and one output, the resulting solution is returned with multiple solutions for a nonlinear equation. For a system of equations and an equal number of outputs, the results are sorted alphabetically and assigned to the outputs. For a system of equations and a single output, a structure containing the solutions is returned.

For both a single equation and a system of equations, numeric solutions are returned if symbolic solutions cannot be determined.

**Examples** Solve the quadratic equation:

```
syms a b c x;  
solve('a*x^2 + b*x + c')
```

The result is:

```
ans =  
-(b + (b^2 - 4*a*c)^(1/2))/(2*a)  
-(b - (b^2 - 4*a*c)^(1/2))/(2*a)
```

Solve the quadratic equation for the variable b:

```
syms a b c x;  
solve('a*x^2 + b*x + c', 'b')
```

The result is:

```
ans =  
-(a*x^2 + c)/x
```

Solve the system of equations:

```
syms x;  
S = solve('x + y = 1', 'x - 11*y = 5');  
S = [S.x S.y]
```

The result is:

```
S =  
[ 4/3, -1/3]
```

The statement

```
syms a u v;  
A = solve('a*u^2 + v^2', 'u - v = 1', 'a^2 - 5*a + 6')
```

returns

```
A =  
a: [4x1 sym]  
u: [4x1 sym]
```



v: [4x1 sym]

To see the elements of A, enter:

Aa = A.a

Au = A.u

Av = A.v

Aa =

3

2

2

3

Au =

$(3^{(1/2)}i)/4 + 1/4$

$(2^{(1/2)}i)/3 + 1/3$

$1/3 - (2^{(1/2)}i)/3$

$1/4 - (3^{(1/2)}i)/4$

Av =

$(3^{(1/2)}i)/4 - 3/4$

$(2^{(1/2)}i)/3 - 2/3$

-  $(2^{(1/2)}i)/3 - 2/3$

-  $(3^{(1/2)}i)/4 - 3/4$

## See Also

Arithmetic Operations, dsolve, symvar

# sort

---

**Purpose** Sort symbolic vectors or polynomials

**Syntax** `Y = sort(v)`  
`Y = sort(p)`

**Description** `Y = sort(v)` sorts the elements of a symbolic vector `v` in numerical or lexicographic order.  
`Y = sort(p)` sorts the terms in a polynomial `p` in order of decreasing powers.

**Examples** Sort the elements of a symbolic matrices and vectors:

```
syms a b c d e;  
sort(sym(magic(3)))  
sort([a c e b d])
```

The results are:

```
ans =  
[ 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

```
ans =  
[ a, b, c, d, e]
```

Sort the terms of the polynomial:

```
syms a b c d e x;  
sort([a c e b d]*x.^(0:4).')
```

The result is:

```
ans =  
d*x^4 + b*x^3 + e*x^2 + c*x + a
```

**See Also** `sym2poly`, `coeffs`

**Purpose** Rewrite symbolic expression in terms of common subexpressions

**Syntax** [Y, SIGMA] = subexpr(X, SIGMA)  
 [Y, SIGMA] = subexpr(X, 'SIGMA')

**Description** [Y, SIGMA] = subexpr(X, SIGMA) or [Y, SIGMA] = subexpr(X, 'SIGMA') rewrites the symbolic expression X in terms of its common subexpressions.

**Examples** The statements

```
h = solve('a*x^3+b*x^2+c*x+d = 0');
[r,s] = subexpr(h,'s')
```

return the rewritten expression for **t** in **r** in terms of a common subexpression, which is returned in **s**:

```
r =
      s^(1/3) - b/(3*a) - (c/(3*a) - b^2/(9*a^2))/s^(1/3)
(c/(3*a) - b^2/(9*a^2))/(2*s^(1/3)) - s^(1/3)/2 - b/(3*a) - (3^(1/2)*i*(s^(1/3) + ...
(c/(3*a) - b^2/(9*a^2))/s^(1/3))/2
(c/(3*a) - b^2/(9*a^2))/(2*s^(1/3)) - s^(1/3)/2 - b/(3*a) + (3^(1/2)*i*(s^(1/3) + ...
(c/(3*a) - b^2/(9*a^2))/s^(1/3))/2

s =
((d/(2*a) + b^3/(27*a^3) - (b*c)/(6*a^2))^2 + (c/(3*a) - b^2/(9*a^2))^3)^(1/2) - ...
b^3/(27*a^3) - d/(2*a) + (b*c)/(6*a^2)
```

**See Also** pretty, simple, subs

# subs

---

**Purpose** Symbolic substitution in symbolic expression or matrix

**Syntax**

```
R = subs(S)
R = subs(S, new)
R = subs(S, old, new)
```

**Description**

`R = subs(S)` replaces all occurrences of variables in the symbolic expression `S` with values obtained from the calling function, or the MATLAB workspace.

`R = subs(S, new)` replaces the default symbolic variable in `S` with `new`.

`R = subs(S, old, new)` replaces `old` with `new` in the symbolic expression `S`. `old` is a symbolic variable or a string representing a variable name. `new` is a symbolic or numeric variable or expression. That is, `R = subs(S,old,new)` evaluates `S` at `old = new`. The substitution is first attempted as a MATLAB expression resulting in the computation being done in double precision arithmetic if all the values in `new` are double precision. Convert the new values to `sym` to ensure symbolic or variable precision arithmetic.

If `old` and `new` are vectors or cell arrays of the same size and type, each element of `old` is replaced by the corresponding element of `new`. If `S` and `old` are scalars and `new` is an array or cell array, the scalars are expanded to produce an array result. If `new` is a cell array of numeric matrices, the substitutions are performed elementwise (i.e., `subs(x*y, {x, y}, {A, B})` returns `A.*B` when `A` and `B` are numeric).

If `subs(s,old,new)` does not change `s`, `subs(s,new,old)` is tried. This provides backwards compatibility with previous versions and eliminates the need to remember the order of the arguments. `subs(s,old,new,0)` does not switch the arguments if `s` does not change.

---

**Note** If  $A$  is a matrix, the command `subs(S, x, A)` replaces all occurrences of the variable  $x$  in the symbolic expression  $S$  with the matrix  $A$ , and replaces the constant term in  $S$  with the constant times a matrix of all ones. To evaluate  $S$  in the matrix sense, use the command `polyvalm(sym2poly(S), A)`, which replaces the constant term with the constant times an identity matrix.

---

## Examples

### Single Input

Suppose  $a = 980$  and  $C2 = 3$  exist in the workspace.

The statement

```
y = dsolve('Dy = -a*y')
```

produces

```
y =  
C2/exp(a*t)
```

Then the statements

```
a = 980; C2 = 3; subs(y)
```

produce

```
ans =  
3/exp(980*t)
```

### Single Substitution

```
syms a b;  
subs(a + b, a, 4)
```

returns

```
ans =
```

```
b + 4
```

## Multiple Substitutions

```
syms a b;  
subs(cos(a) + sin(b), {a, b}, {sym('alpha'), 2})
```

returns

```
ans =  
sin(2) + cos(alpha)
```

## Scalar Expansion Case

```
syms t;  
subs(exp(a*t), 'a', -magic(2))
```

returns

```
ans =  
[ 1/exp(t), 1/exp(3*t)]  
[ 1/exp(4*t), 1/exp(2*t)]
```

## Multiple Scalar Expansion

```
syms x y;  
subs(x*y, {x, y}, {[0 1; -1 0], [1 -1; -2 1]})
```

returns

```
ans =  
 0   -1  
 2    0
```

## See Also

[simplify](#), [subexpr](#)

**Purpose**

Compute singular value decomposition of symbolic matrix

**Syntax**

```
sigma = svd(A)
sigma = svd(vpa(A))
[U, S, V] = svd(A)
[U, S, V] = svd(vpa(A))
```

**Description**

`sigma = svd(A)` returns a symbolic vector containing the singular values of a symbolic matrix `A`. With symbolic inputs, `svd` does not accept complex values as inputs.

`sigma = svd(vpa(A))` returns a vector with the numeric singular values using variable precision arithmetic.

`[U, S, V] = svd(A)` and `[U, S, V] = svd(vpa(A))` return numeric unitary matrices `U` and `V` with the columns containing the singular vectors and a diagonal matrix `S` containing the singular values. The matrices satisfy  $A = U*S*V'$ . The `svd` command does not compute symbolic singular vectors. With multiple outputs, `svd` does not accept complex values as inputs.

**Examples**

Compute the symbolic and numeric singular values and the numeric singular vectors of the following magic square:

```
digits(5)
A = sym(magic(4));
svd(A)
svd(vpa(A))
[U, S, V] = svd(A)
```

The results are:

```
ans =
      0
  2*5^(1/2)
  8*5^(1/2)
      34
```

```
ans =  
      34.0  
      17.889  
      4.4721  
 2.8024*10^(-7)  
  
U =  
[ 0.5, 0.67082, 0.5, 0.22361]  
[ 0.5, -0.22361, -0.5, 0.67082]  
[ 0.5, 0.22361, -0.5, -0.67082]  
[ 0.5, -0.67082, 0.5, -0.22361]  
  
S =  
[ 34.0, 0, 0, 0]  
[ 0, 17.889, 0, 0]  
[ 0, 0, 4.4721, 0]  
[ 0, 0, 0, 0]  
  
V =  
[ 0.5, 0.5, 0.67082, 0.22361]  
[ 0.5, -0.5, -0.22361, 0.67082]  
[ 0.5, -0.5, 0.22361, -0.67082]  
[ 0.5, 0.5, -0.67082, -0.22361]
```

## See Also

[digits](#) | [eig](#) | [inv](#) | [vpa](#)



**Purpose**

Symbolic numbers, variables, and objects

**Syntax**

```
S = sym(A)
x = sym('x')
x = sym('x', 'real')
k = sym('k', 'positive')
x = sym('x', 'clear')
S = sym(A, flag)
```

**Description**

`S = sym(A)` constructs an object `S`, of class `'sym'`, from `A`. If the input argument is a string, the result is a symbolic number or variable. If the input argument is a numeric scalar or matrix, the result is a symbolic representation of the given numeric values.

`x = sym('x')` creates the symbolic variable with name `'x'` and stores the result in `x`.

`x = sym('x', 'real')` also assumes that `x` is real, so that `conj(x)` is equal to `x`. `alpha = sym('alpha')` and `r = sym('Rho', 'real')` are other examples.

Similarly, `k = sym('k', 'positive')` makes `k` a positive (real) variable.

`x = sym('x', 'clear')` makes `x` a purely formal variable with no additional properties (i.e., ensures that `x` is neither real nor positive). See also the reference pages on `syms`. For compatibility with previous versions of the software, `x = sym('x', 'unreal')` has exactly the same effect as `x = sym('x', 'clear')`.

Statements like `pi = sym('pi')` and `delta = sym('1/10')` create symbolic numbers that avoid the floating-point approximations inherent in the values of `pi` and `1/10`. The `pi` created in this way temporarily replaces the built-in numeric function with the same name.

`S = sym(A, flag)` where `flag` is one of `'r'`, `'d'`, `'e'`, or `'f'`, converts a numeric scalar or matrix to symbolic form. The technique for converting floating-point numbers is specified by the optional second argument, which can be `'f'`, `'r'`, `'e'` or `'d'`. The default is `'r'`.

'f' stands for "floating-point." All values are represented in the form  $N \cdot 2^e$  or  $-N \cdot 2^e$ , where  $N$  and  $e$  are integers,  $N \geq 0$ . For example, `sym(1/10, 'f')` is `3602879701896397/36028797018963968`.

'r' stands for "rational." Floating-point numbers obtained by evaluating expressions of the form  $p/q$ ,  $p \cdot \pi/q$ ,  $\sqrt{p}$ ,  $2^q$ , and  $10^q$  for modest sized integers  $p$  and  $q$  are converted to the corresponding symbolic form. This effectively compensates for the roundoff error involved in the original evaluation, but may not represent the floating-point value precisely. If no simple rational approximation can be found, an expression of the form  $p \cdot 2^q$  with large integers  $p$  and  $q$  reproduces the floating-point value exactly. For example, `sym(4/3, 'r')` is `'4/3'`, but `sym(1+sqrt(5), 'r')` is `7286977268806824*2^(-51)`.

'e' stands for "estimate error." The 'r' form is supplemented by a term involving the variable 'eps', which estimates the difference between the theoretical rational expression and its actual floating-point value. For example, `sym(3*pi/4, 'e')` is `3*pi/4*(1+3143276*eps/65)`.

'd' stands for "decimal." The number of digits is taken from the current setting of `digits` used by `vpa`. Fewer than 16 digits loses some accuracy, while more than 16 digits may not be warranted. For example, with `digits(10)`, `sym(4/3, 'd')` is `1.333333333`, while with `digits(20)`, `sym(4/3, 'd')` is `1.3333333333333332593`, which does not end in a string of 3s, but is an accurate decimal representation of the floating-point number nearest to  $4/3$ .

## See Also

`digits`, `double`, `findsym`, `reset`, `syms`, `symvar`  
`eps` in the online MATLAB Function Reference

**Purpose** Choose symbolic engine

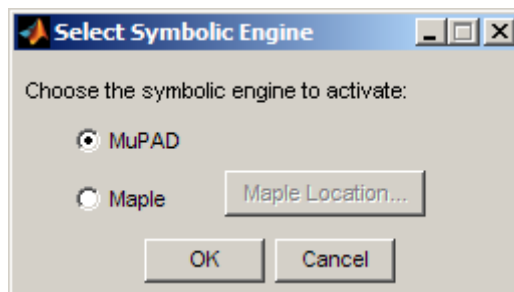
**Syntax**  
`symengine`  
`s = symengine`

**Description** `symengine` brings up a GUI for choosing the MATLAB symbolic engine: a MuPAD engine or a Maple engine.

`s = symengine` returns the currently active symbolic engine.

**Example** `symengine`

brings up the following GUI:



To see which symbolic computation engine is currently active, enter:

```
s = symengine
```

The result is:

```
s =  
MuPAD symbolic engine
```

**See Also** “Choosing a Maple or MuPAD Engine” on page 4-34

**Purpose**            Shortcut for constructing symbolic objects

**Syntax**            `syms arg1 arg2 ...`  
`syms arg1 arg2 ... real`  
`syms arg1 arg2 ... clear`  
`syms arg1 arg2 ... positive`

**Description**      `syms arg1 arg2 ...` is a shortcut for

```
arg1 = sym('arg1');  
arg2 = sym('arg2'); ...
```

`syms arg1 arg2 ... real` is a shortcut for

```
arg1 = sym('arg1','real');  
arg2 = sym('arg2','real'); ...
```

`syms arg1 arg2 ... clear` is a shortcut for

```
arg1 = sym('arg1','clear');  
arg2 = sym('arg2','clear'); ...
```

`syms arg1 arg2 ... positive` is a shortcut for

```
arg1 = sym('arg1','positive');  
arg2 = sym('arg2','positive'); ...
```

Each input argument must begin with a letter and can contain only alphanumeric characters. For compatibility with previous versions of the software, `syms arg1 arg2 ... unreal` has exactly the same effect as `syms arg1 arg2 ... clear`.

In functions and scripts, do not use the `syms` command to create symbolic variables with the same names as MATLAB functions. For these names MATLAB does not create symbolic variables, but keeps the names assigned to the functions. If you want to create a symbolic variable with the same name as some MATLAB function inside a function or a script, use the `sym` command. For example:

```
alpha = sym('alpha')
```

## Examples

`syms x y real` is equivalent to

```
x = sym('x','real');  
y = sym('y','real');
```

To clear the symbolic objects `x` and `y` of 'real' status, type

```
syms x y clear
```

Note that `clear x` will *not* clear the symbolic object of its 'real' status. You can achieve this using

- `syms x clear` to remove the 'real' status from `x` without affecting any other symbolic variables.
- `reset(symengine)` resets the MuPAD engine.
- `clear all` clears all objects in the MATLAB workspace and resets the MuPAD engine.

## See Also

`findsym`, `reset`, `sym`, `symvar`

“Clearing Assumptions and Resetting the Symbolic Engine” on page 4-29

# sym2poly

---

**Purpose** Symbolic-to-numeric polynomial conversion

**Syntax** `c = sym2poly(s)`

**Description** `c = sym2poly(s)` returns a row vector containing the numeric coefficients of a symbolic polynomial. The coefficients are ordered in descending powers of the polynomial's independent variable. In other words, the vector's first entry contains the coefficient of the polynomial's highest term; the second entry, the coefficient of the second highest term; and so on.

**Examples** The command

```
syms x u v
sym2poly(x^3 - 2*x - 5)
```

returns

```
ans =
     1     0    -2    -5
```

The command

```
sym2poly(u^4 - 3 + 5*u^2)
```

returns

```
ans =
     1     0     5     0    -3
```

and the command

```
sym2poly(sin(pi/6)*v + exp(1)*v^2)
```

returns

```
ans =
 2.7183    0.5000     0
```

**See Also**

poly2sym, subs, sym, polyval in the online MATLAB Function Reference

# symsum

---

**Purpose** Evaluate symbolic sum of series

**Syntax**

```
r = symsum(expr)
r = symsum(expr, v)
r = symsum(expr, a, b)
r = symsum(expr, v, a, b)
```

**Description**

`r = symsum(expr)` evaluates the sum of the symbolic expression `expr` with respect to the default symbolic variable `defaultVar` determined by `symvar`. The value of the default variable changes from 0 to `defaultVar - 1`.

`r = symsum(expr, v)` evaluates the sum of the symbolic expression `expr` with respect to the symbolic variable `v`. The value of the variable `v` changes from 0 to `v - 1`.

`r = symsum(expr, a, b)` evaluates the sum of the symbolic expression `expr` with respect to the default symbolic variable `defaultVar` determined by `symvar`. The value of the default variable changes from `a` to `b`.

`r = symsum(expr, v, a, b)` evaluates the sum of the symbolic expression `expr` with respect to the symbolic variable `v`. The value of the default variable changes from `a` to `b`.

**Examples** Evaluate the sum of the following symbolic expressions `k` and `k^2`:

```
syms k
symsum(k)
symsum(k^2)
```

The results are

```
ans =
k^2/2 - k/2
```

```
ans =
k^3/3 - k^2/2 + k/6
```



Evaluate the sum of the following expression specifying the limits:

```
symsum(k^2, 0, 10)
```

The result is

```
ans =
385
```

---

Evaluate the sum of the following multivariable expression with respect to k:

```
syms x;
symsum(x^k/sym('k!'), k, 0, inf)
```

The result is

```
ans =
exp(x)
```

**See Also**

[int](#) | [syms](#) | [symvar](#)

**How To**

- “Symbolic Summation” on page 3-19

# symvar

---

**Purpose** Find symbolic variables in symbolic expression or matrix

**Syntax** `symvar(s)`  
`symvar(s,n)`

**Description** `symvar(s)` returns a vector containing all the symbolic variables in `s`. The variables are returned in the alphabetical order with uppercase letters preceding lowercase letters. If there are no symbolic variables in `s`, then `symvar` returns the empty vector. `symvar` does not consider the constants `pi`, `i`, and `j` to be variables.

`symvar(s,n)` returns a vector containing the `n` symbolic variables in `s` that are alphabetically closest to 'x':

- 1 The variables are sorted by the first letter in their names. The ordering is x y w z v u ... a X Y W Z V U ... A. The name of a symbolic variable cannot begin with a number.
- 2 For all subsequent letters, the ordering is alphabetical, with all uppercase letters having precedence over lowercase: 0 1 ... 9 A B ... Z a b ...z.

---

**Note** `symvar(s)` can return variables in different order than `symvar(s,n)`.

---

## Examples

```
syms wa wb wx yx ya yb
f = wa + wb + wx + ya + yb + yx;
symvar(f)
```

The result is:

```
ans =
[ wa, wb, wx, ya, yb, yx]
```

```
syms x y z a b
```

```
w = x^2/(sin(3*y - b));
symvar(w)
```

The result is:

```
ans =
 [ b, x, y]

symvar(w, 3)
```

The result is:

```
ans =
 [ x, y, b]
```

`symvar(s,1)` returns the variable closest to `x`. When performing differentiation, integration, substitution or solving equations, MATLAB uses this variable as a default variable.

```
syms v z
g = v + z;
symvar(g, 1)
```

The result is:

```
ans =
 z

syms aaa aab
g = aaa + aab;
symvar(g, 1)
```

The result is:

```
ans =
 aaa

syms X1 x2 xa xb
g = X1 + x2 + xa + xb;
```

# symvar

---

```
symvar(g, 1)
```

The result is:

```
ans =  
x2
```

## See Also

findsym, sym, syms

---

<b>Purpose</b>	Taylor series expansion
<b>Syntax</b>	<code>taylor(f)</code> <code>taylor(f, n)</code> <code>taylor(f, a)</code> <code>taylor(f, n, v)</code> <code>taylor(f, n, v, a)</code>
<b>Description</b>	<p><code>taylor(f)</code> returns the fifth order Maclaurin polynomial approximation to <code>f</code>.</p> <p><code>taylor(f, n)</code> returns the <math>(n-1)</math>-order Maclaurin polynomial approximation to <code>f</code>. Here <code>n</code> is a positive integer.</p> <p><code>taylor(f, a)</code> returns the fifth order Taylor series approximation to <code>f</code> about point <code>a</code>. Here <code>a</code> is a real number. If <code>a</code> is a positive integer or if you want to change the expansion order, use <code>taylor(f, n, a)</code> to specify the base point and the expansion order.</p> <p><code>taylor(f, n, v)</code> returns the <math>(n-1)</math>-order Maclaurin polynomial approximation to <code>f</code>, where <code>f</code> is a symbolic expression representing a function and <code>v</code> specifies the independent variable in the expression. <code>v</code> can be a string or symbolic variable.</p> <p><code>taylor(f, n, v, a)</code> returns the Taylor series approximation to <code>f</code> about <code>a</code>. The argument <code>a</code> can be a numeric value, a symbol, or a string representing a numeric value or an unknown. If <code>a</code> is a symbol or a string, do not omit <code>v</code>.</p> <p>If <code>a</code> is neither an integer nor a symbol or a string, you can supply the arguments <code>n</code>, <code>v</code>, and <code>a</code> in any order. <code>taylor</code> determines the purpose of the arguments from their position and type.</p> <p>You also can omit any of the arguments <code>n</code>, <code>v</code>, and <code>a</code>. If you do not specify <code>v</code>, <code>taylor</code> uses <code>symvar</code> to determine the function's independent variable. <code>n</code> defaults to 6, and <code>a</code> defaults to 0.</p> <p>The following expression present the Taylor series for an analytic function <math>f(x)</math> about the base point <math>x=a</math>:</p>

$$f(x) = \sum_{m=0}^{\infty} (x-a)^m \cdot \frac{f^{(m)}(a)}{m!}$$

## Examples

The following table describes the various uses of the `taylor` command and its relation to Taylor and Maclaurin series. Before using the `taylor` command, define the function you want to expand. For example:

```
syms x
f = exp(x^2);
```

Mathematical Operation	MATLAB Operation
$\sum_{m=0}^5 x^m \cdot \frac{f^{(m)}(0)}{m!}$	<code>taylor(f)</code>
$\sum_{m=0}^{n-1} x^m \cdot \frac{f^{(m)}(0)}{m!}$ <p><i>n</i> is a positive integer</p>	<code>taylor(f,n)</code> <i>n</i> is a positive integer.
$\sum_{m=0}^5 (x-a)^m \cdot \frac{f^{(m)}(a)}{m!}$ <p><i>a</i> is a real number</p>	<code>taylor(f,a)</code> <i>a</i> is a real number.
$\sum_{m=0}^{n-1} (x-a)^m \cdot \frac{f^{(m)}(a)}{m!}$ <p><i>n</i> is a positive integer and <i>a</i> is real. Also, <i>a</i> can be a positive integer.</p>	<code>taylor(f,n,a)</code> <i>a</i> is real and <i>n</i> is a positive integer.

In the case where `f` is a function of two or more variables (`f=f(x,y,...)`), there is an additional parameter that allows you to

select the variable for the Taylor expansion. Look at this table for illustrations of this feature.

Mathematical Operation	MATLAB Operation
$\sum_{m=0}^5 \frac{y^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=0}$	taylor(f, y)
$\sum_{m=0}^{n-1} \frac{y^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=0}$ <p><i>n</i> is a positive integer</p>	taylor(f, y, n) or taylor(f, n, y) <i>n</i> is a positive integer.
$\sum_{m=0}^5 \frac{(y-a)^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=a}$ <p><i>a</i> is real</p>	taylor(f, y, a) <i>a</i> is real.
$\sum_{m=0}^{n-1} \frac{(y-a)^m}{m!} \cdot \frac{\partial^m}{\partial y^m} f(x, y) \Big _{y=a}$ <p><i>a</i> is real and <i>n</i> is a positive integer</p>	taylor(f, n, y, a) <i>a</i> is real and <i>n</i> is a positive integer.

**See Also**

symvar, taylorTool

# taylortool

---

**Purpose** Taylor series calculator

**Syntax** `taylortool`  
`taylortool('f')`

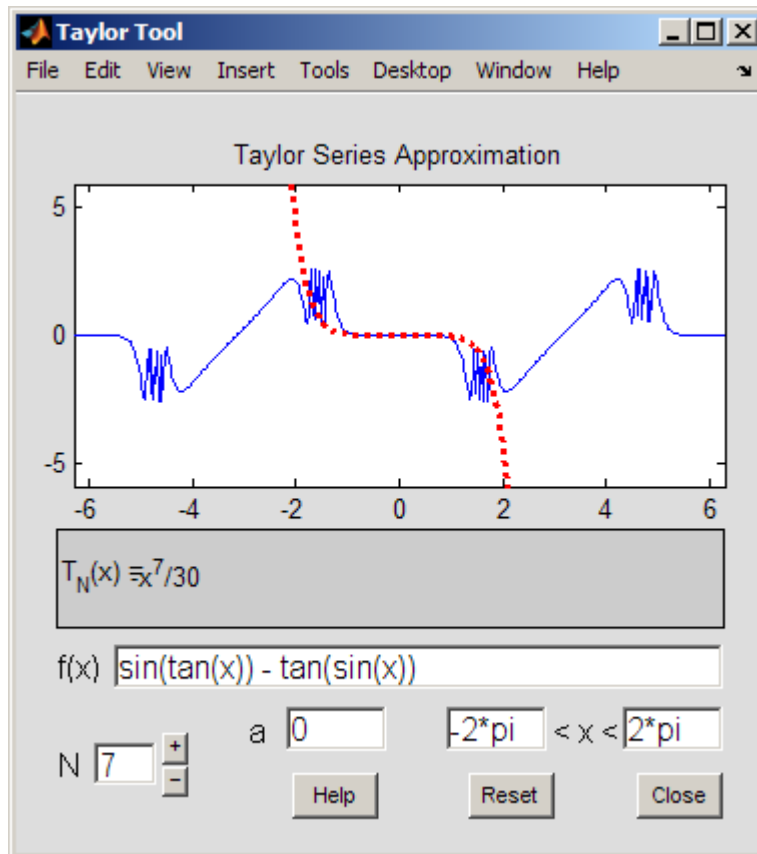
**Description** `taylortool` initiates a GUI that graphs a function against the Nth partial sum of its Taylor series about a base point  $x = a$ . The default function, value of N, base point, and interval of computation for `taylortool` are  $f = x \cdot \cos(x)$ ,  $N = 7$ ,  $a = 0$ , and  $[-2\pi, 2\pi]$ , respectively.

`taylortool('f')` initiates the GUI for the given expression  $f$ .

## Examples

```
taylortool('sin(tan(x)) - tan(sin(x))')
```





**See Also** funtool, rsums

**Purpose** Return lower triangular part of symbolic matrix

**Syntax** `tril(A)`  
`tril(A, k)`

**Description** `tril(A)` returns a triangular matrix that retains the lower part of the matrix A. The upper triangle of the resulting matrix is padded with zeros.

`tril(A, k)` returns a matrix that retains the elements of A on and below the  $k$ -th diagonal. The elements above the  $k$ -th diagonal equal to zero. The values  $k = 0$ ,  $k > 0$ , and  $k < 0$  correspond to the main, superdiagonals, and subdiagonals, respectively.

**Examples** Display the matrix retaining only the lower triangle of the original symbolic matrix:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
tril(A)
```

The result is:

```
ans =
[ a, 0, 0]
[ 1, 2, 0]
[ a + 1, b + 2, c + 3]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and below the first superdiagonal:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
tril(A, 1)
```

The result is:

```
ans =  
[      a,      b,      0]  
[      1,      2,      3]  
[ a + 1, b + 2, c + 3]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and below the first subdiagonal:

```
syms a b c  
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];  
tril(A, -1)
```

The result is:

```
ans =  
[      0,      0, 0]  
[      1,      0, 0]  
[ a + 1, b + 2, 0]
```

## See Also

[diag](#) | [triu](#)

# triu

---

**Purpose** Return upper triangular part of symbolic matrix

**Syntax** `triu(A)`  
`triu(A, k)`

**Description** `triu(A)` returns a triangular matrix that retains the upper part of the matrix A. The lower triangle of the resulting matrix is padded with zeros.

`triu(A, k)` returns a matrix that retains the elements of A on and above the  $k$ -th diagonal. The elements below the  $k$ -th diagonal equal to zero. The values  $k = 0$ ,  $k > 0$ , and  $k < 0$  correspond to the main, superdiagonals, and subdiagonals, respectively.

**Examples** Display the matrix retaining only the upper triangle of the original symbolic matrix:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
triu(A)
```

The result is:

```
ans =
[ a, b,    c]
[ 0, 2,    3]
[ 0, 0, c + 3]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and above the first superdiagonal:

```
syms a b c
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];
triu(A, 1)
```

The result is:

```
ans =  
[ 0, b, c]  
[ 0, 0, 3]  
[ 0, 0, 0]
```

---

Display the matrix that retains the elements of the original symbolic matrix on and above the first subdiagonal:

```
syms a b c  
A = [a b c; 1 2 3; a + 1 b + 2 c + 3];  
triu(A, -1)
```

The result is:

```
ans =  
[ a,      b,      c]  
[ 1,      2,      3]  
[ 0, b + 2, c + 3]
```

## See Also

[diag](#) | [tril](#)

# uint8, uint16, uint32, uint64

**Purpose** Convert symbolic matrix to unsigned integers

**Syntax** uint8(S)  
uint16(S)  
uint32(S)  
uint64(S)

**Description** uint8(S) converts a symbolic matrix S to a matrix of unsigned 8-bit integers.  
uint16(S) converts S to a matrix of unsigned 16-bit integers.  
uint32(S) converts S to a matrix of unsigned 32-bit integers.  
uint64(S) converts S to a matrix of unsigned 64-bit integers.

---

**Note** The output of uint8, uint16, uint32, and uint64 does not have type symbolic.

---

The following table summarizes the output of these four functions.

Function	Output Range	Output Type	Bytes per Element	Output Class
uint8	0 to 255	Unsigned 8-bit integer	1	uint8
uint16	0 to 65,535	Unsigned 16-bit integer	2	uint16
uint32	0 to 4,294,967,295	Unsigned 32-bit integer	4	uint32
uint64	0 to 18,446,744,073,709,551,615	Unsigned 64-bit integer	8	uint64

**See Also** sym, vpa, single, double, int8, int16, int32, int64

- Purpose** Find unique elements of symbolic matrices or their numeric approximations
- Syntax** `unique(A)`  
`unique(vpa(A))`
- Description** `unique(A)` for the symbolic matrix `A` returns the same values as in `A`, but with no repetitions. The `unique` command also sorts the elements of the resulting vector alphabetically. If the resulting vector contains symbolic numbers, the command sorts them in ascending order.
- `unique(vpa(A))` for the symbolic matrix `A` returns the numeric approximations of the elements of `A`, but with no repetitions. The `unique` command also sorts the elements of the resulting vector in ascending order.
- Examples** Sort the elements of a symbolic vector excluding duplicates:

```
syms a b c d e x;  
unique([a b c 5 2 b c d 3 b e x x])
```

The result is:

```
ans =  
[ 2, 3, 5, a, b, c, d, e, x]
```

---

Sort the elements of the symbolic Hilbert matrix:

```
H = sym(hilb(5));  
unique(H)
```

The result is a column vector of unique elements:

```
ans =  
1/9  
1/8
```





**Purpose**

Variable precision arithmetic

**Syntax**

```
R = vpa(A)
R = vpa(A, d)
```

**Description**

`R = vpa(A)` uses variable-precision arithmetic (VPA) to compute each element of `A` to `d` decimal digits of accuracy, where `d` is the current setting of `digits`. Each element of the result is a symbolic expression.

`R = vpa(A, d)` uses `d` digits, instead of the current setting of `digits`.

**Examples**

The statements

```
digits(25)
q = vpa(sin(sym('pi')/6))
p = vpa(pi)
w = vpa('(1+sqrt(5))/2')
```

return

```
q =
0.5
```

```
p =
3.141592653589793238462643
```

```
w =
1.618033988749894848204587
```

`vpa pi 75` computes  $\pi$  to 75 digits.

The statements

```
A = vpa(hilb(2),25)
B = vpa(hilb(2),5)
```

return



**Purpose** Compute the Riemann zeta function

**Syntax** `Y = zeta(X)`  
`Y = zeta(n, X)`

**Description** `Y = zeta(X)` evaluates the zeta function at the elements of `X`, a numeric matrix, or a symbolic matrix. The zeta function is defined by

$$\zeta(w) = \sum_{k=1}^{\infty} \frac{1}{k^w}$$

`Y = zeta(n, X)` returns the `n`-th derivative of `zeta(X)`.

**Examples** Compute the Riemann zeta function for the number:

```
zeta(1.5)
```

The result is:

```
ans =
    2.6124
```

Compute the Riemann zeta function for the matrix:

```
zeta(1.2:0.1:2.1)
```

The result is:

```
ans =
Columns 1 through 6
    5.5916    3.9319    3.1055    2.6124    2.2858    2.0543
Columns 7 through 10
    1.8822    1.7497    1.6449    1.5602
```

Compute the Riemann zeta function for the matrix of the symbolic expressions:

```
syms x y;  
zeta([x 2; 4 x + y])
```

The result is:

```
ans =  
[ zeta(x),      pi^2/6]  
[ pi^4/90, zeta(x + y)]
```

Differentiate the Riemann zeta function:

```
diff(zeta(x), x, 3)
```

The result is:

```
ans =  
zeta(x, 3)
```

**Purpose** $z$ -transform**Syntax**

$F = \text{ztrans}(f)$   
 $F = \text{ztrans}(f, w)$   
 $F = \text{ztrans}(f, k, w)$

**Description**

$F = \text{ztrans}(f)$  is the  $z$ -transform of the scalar symbol  $f$  with default independent variable  $n$ . The default return is a function of  $z$ .

$$f = f(n) \Rightarrow F = F(z)$$

The  $z$ -transform of  $f$  is defined as

$$F(z) = \sum_0^{\infty} \frac{f(n)}{z^n}$$

where  $n$  is  $f$ 's symbolic variable as determined by `symvar`. If  $f = f(z)$ , then `ztrans(f)` returns a function of  $w$ .

$$F = F(w)$$

$F = \text{ztrans}(f, w)$  makes  $F$  a function of the symbol  $w$  instead of the default  $z$ .

$$F(w) = \sum_0^{\infty} \frac{f(n)}{w^n}$$

$F = \text{ztrans}(f, k, w)$  takes  $f$  to be a function of the symbolic variable  $k$ .

$$F(w) = \sum_0^{\infty} \frac{f(k)}{w^k}$$

## Examples

Z-Transform	MATLAB Operation
$f(n) = n^4$ $Z[f] = \sum_{n=0}^{\infty} f(n)z^{-n}$ $= \frac{z(z^3 + 11z^2 + 11z + 1)}{(z-1)^5}$	<pre>syms n; f = n^4; ztrans(f)  ans = (z^4 + 11*z^3 + 11*z^2 + z)/(z - 1)^5</pre>
$g(z) = a^z$ $Z[g] = \sum_{z=0}^{\infty} g(z)w^{-z}$ $= \frac{w}{w-a}$	<pre>syms a z; g = a^z; ztrans(g)  ans = -w/(a - w)</pre>
$f(n) = \sin(an)$ $Z[f] = \sum_{n=0}^{\infty} f(n)w^{-n}$ $= \frac{w \sin a}{1 - 2w \cos a + w^2}$	<pre>syms a n w; f = sin(a*n); ztrans(f, w)  ans = (w*sin(a))/(w^2 - 2*cos(a)*w + 1)</pre>

## See Also

fourier, iztrans, laplace

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